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Comparing mean variance tests with stochastic dominance tests when assessing international portfolio diversification benefits

Thomas O. Meyer[a], Xiao-Ming Li[b], Lawrence C. Rose[b,*]

[a]Department of Marketing and Finance, Southeastern Louisiana University, Hammond, LA 70402, USA
[b]Department of Commerce, Massey University, Albany Campus, Auckland, New Zealand

Abstract

Stochastic dominance is theoretically superior to mean-variance (MV) analysis because it considers the entire return distribution and is based on minimally restrictive assumptions regarding investor motives. This study uses stochastic dominance to examine whether adding internationally based assets to a wholly domestic portfolio generates diversification benefits for an investor. In contrast to previous MV findings, a New Zealand-only portfolio stochastically dominates four internationally diversified portfolios across all periods considered. Similarly, the least internationally diversified portfolio persistently dominates more diversified counterparts. Within-portfolio analysis shows that in the Asian Crisis period, the least risky or lowest return weighting schemes dominate those with greater risk and/or higher return characteristics. © 2005 Academy of Financial Services. All rights reserved.

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1. Introduction

A tenet of modern financial theory related to the pioneering works of Markowitz (1952, 1959) is that an investor should construct a diversified portfolio of investments to achieve the most favorable tradeoff between risk and return. A great deal of research has been done on the diversification benefits achievable via international investment. However, as world

* Corresponding author. Tel.: +1-649-414-0800; fax: +1-649-441-8177.
E-mail address: l.c.rose@massey.ac.nz (L.C. Rose).

capital markets become increasingly more integrated, a question arises as to whether an investor can obtain international diversification gains.

Another critical and important issue regarding international diversification is the appropriate framework for assessing benefits. Traditionally, empirical portfolio analysis has focused on the mean-variance (MV) characteristics of assets. The Sharpe (1966) ratio is commonly used to quantify the risk-return tradeoff. But, use of MV analysis and the Sharpe ratio could end up misleading an investor as the Sharpe ratio can rank investments in an illogical manner when an investment's excess return is negative. Further, investment horizons may play a role in misleading rankings. Hodges, Taylor, and Yoder (1997, 2002) indicate that investor holding periods must match the intervals used to calculate Sharpe or Treynor (1965) ratios or the ratios yield invalid portfolio rankings.

Alternatively, Porter, Wart, and Ferguson (1973, pg. 71) state that stochastic dominance (SD) "has been shown to be theoretically superior to all of these 'moment methods.'" Kuosmanen (2001) suggests that SD is attractive because it is effectively non-parametric as no explicit specification of a utility function or probability distribution functional form is required. Additionally, it considers the entire distribution, not selected moments. So it may be more robust, especially when market volatility is high.

This research uses SD analysis methods to assess international portfolio diversification benefits. Then, SD analysis is compared to the results of mean-variance analysis to assess any complementarity of approaches and the robustness of SD findings.

Many studies of the benefits of international diversification employ historical data when forming portfolios which effectively assumes perfect foresight. Therefore, it is not surprising to see that domestic portfolios dominate those that are diversified internationally. The perfect foresight issue is overcome here by determining efficient portfolio weighting schemes (PWS) in an ex-ante period. Exactly the same PWS are used in conjunction with asset returns in two subsequent, non-overlapping, ex-post periods in out-of-sample tests. These results may then be contrasted with similar tests like those in Jorion (1985), Grauer and Hakkansson (1987), and Meyer and Rose (2003).

Several secondary issues of importance to investors are also addressed. First, does evidence of SD for a given portfolio in one period of analysis carry over into a subsequent, non-overlapping period? Second, does a dominant high return or high risk portfolio weighting scheme within a given portfolio continue to dominate a lower return or lower risk scheme in a subsequent period? Third, on the basis of the portfolios analyzed, which of the different types of SD best detect dominance? Finally, are there technical aspects regarding calculation of the Sharpe ratio that call into question its validity as a measure of the risk-return tradeoff in MV analysis itself?

The results of stochastic dominance tests comparing New-Zealand-only portfolios to internationally diversified portfolios indicate only third-order SD (TSD) tests exhibit consistent ability to detect dominant portfolios. Based on the TSD tests (and to a lesser extent SSD) NZO portfolios are found to stochastically dominate the four internationally diversified portfolios. Further, the least internationally diverse 10-asset portfolio exhibits SD over the three more diversified 16-asset portfolios. Comparison of low risk or return portfolio weighting schemes to high risk or return schemes within each portfolio in the ex-ante period shows that neither portfolio focus dominates. However, in the two ex-post periods, low risk or return focus-weighting schemes are found to be dominant almost without exception. In contrast, mean-variance analysis using the Sharpe ratio shows that more internationally diverse portfolios offer significantly better risk-return tradeoffs. On the other hand, coefficient of variation (CV) analysis does not generally (only in seven of 40 significant comparisons) indicate benefits from international diversification. In this sense, the CV analysis supports the SD results, while the Sharpe ratio tests do not.

2. Stochastic dominance and international diversification benefits literature

International portfolio diversification has been of interest to investors and academics since early studies by Grubel (1968), Levy and Sarnat (1970), and Solnik (1974) indicated that significant reductions in risk could be obtained by inclusion of international securities. The size of these potential gains varies but can be large as Eiteman, Stonehill, and Moffett (1998) indicate. Chang, Eun, and Kolodny (1995) find that U.S. investors can even obtain these benefits while minimizing transaction costs by investing in closed-end country funds.

Many international portfolio diversification studies have examined the issue from a U.S. viewpoint. However, there are several studies that have considered the issue in an international setting. Odier and Solnik (1993) consider benefits to Japanese, German, and U.K. investors. They conclude that benefits are similar to those of U.S. investors. Eun and Resnick (1994) consider a Japanese viewpoint and conclude that benefits exist, though they are smaller than those enjoyed by U.S. investors. Meyer and Rose (2003) conclude that international diversification can benefit a New Zealand investor and these benefits can mitigate the impact of a major market disturbance.

However, there is doubt that investors can obtain potential gains. Rajan and Friedman (1997) indicate international diversification benefits exist but also conclude market segmentation plays a role in the benefits' existence. Complicating the issue is the fact that integration may be hard to measure. Ammer and Mei (1996) indicate financial and economic links exhibit lags in transmission. Further, Ratner (1992) and Solnik, Boucrelle, and Le Fur (1996) point out market correlations vary over time. This is especially true in times of high market volatility (Longin & Solnik, 1995). Following the Asian Crisis, Tuluca and Zwick (2001) report a dramatic increase in equity return co-movement.

There are a variety of methods used to identify optimal portfolio strategies. One alternative that has been used to evaluate dominant strategies is stochastic dominance. Levy (1973) uses first degree stochastic dominance (FSD) and second degree stochastic dominance (SSD) to see how changes in the size and makeup of the efficient set changes over an investment horizon. He finds, contrary to previous work, that the efficient set decreases with an increase in investment horizon. Hodges and Yoder (1996) also consider investment decisions as the investment horizon lengthens using a stochastic dominance framework. They find no evidence that shares should be preferred over less risky assets like bonds, with the one exception that long-term corporate bonds should be selected over long-term government bonds. Tehranian (1980) concludes that higher degrees of stochastic dominance rules offer consistent rankings and discriminate similar to mean-variance or mean semi-variance rules.

3. Stochastic dominance and hypotheses analyzed

3.1. Description of stochastic dominance

The Markowitz model, where decision makers are assumed to have quadratic utility functions with negative second derivatives has been widely criticized. Criticisms include restrictions on the type of risk preference implied and the normality of the data required. Further, the quadratic utility function implies that beyond some wealth level the investor's marginal utility becomes negative. In contrast, stochastic dominance can be used as an alternative method to examine portfolio construction and rankings.

The stochastic dominance technique uses the entire probability density function rather than a finite number of moments so it can be considered less restrictive. There are no assumptions made concerning the form of the return distributions and not much information on investor preferences is needed to rank alternatives. Higher-order SD tests have increasingly stringent conditions to meet but have higher power of discrimination.

3.2. Testable hypotheses

The major research question considers whether the portfolio of an investor in a small, open economy, represented by New Zealand, can attain diversification benefits through the addition of offshore investments to an initially NZ-only portfolio. It is clear that the results may be sensitive to the choice of New Zealand as the country of analysis. However, the NZ market is chosen for the following reasons. First, it is considered to be a developed market, as opposed to emerging. Therefore, addition of other developed-market unit trusts (UTs) to an initial NZ investment portfolio will not raise the issue of combining assets from developed and emerging markets. Second, a small market in the Asia-Pacific region is employed to examine the effects of the Asian Crisis on portfolio diversification benefits. Third, UT returns formed of assets from New Zealand's major trading partners, for example, Japan, Australia, and Hong Kong could also conceivably be impacted by the Asian crisis. Finally, employing the longest possible trading history is desirable. While other markets, like Singapore, South Korea, or Taiwan, might also serve as the basis for analysis based on the first three criteria, New Zealand had the largest number of unit trusts dating back to 1992 in the data source employed (Datastream). The following hypotheses are tested.

> *Hypothesis 1:* It is expected that internationally diversified portfolios will stochastically dominate the NZ-only portfolio. Further, as the scope of the "internationalization" is increased, increasingly more diversified portfolios are expected to dominate their less diversified counterparts.

A second issue is the question of whether a given portfolio (or a weighting scheme within a portfolio) that stochastically dominates other portfolios (other weighting schemes) in the ex-ante period continue to do so, in later, non-overlapping periods. This gives rise to Hypothesis 2.

> *Hypothesis 2:* A portfolio that is found to stochastically dominate another portfolio in the ex-ante period is expected to also exhibit SD in subsequent periods of analysis.

Similarly, if a specific portfolio weighting scheme (e.g., a high-risk PWS) within a portfolio dominates another (e.g., low-risk) PWS in the formation period, it is also expected to be SD in subsequent periods.

Higher-order SD tests have greater power to discriminate among the choices being compared, but require more restrictions on efficient choice alternatives. Thus, there is a tradeoff between the power of the test and the additional preference assumptions. Given this tradeoff, it is conceivable that lower SD degree(s) may prove to be unable to discriminate between choice alternatives which leads to hypothesis three.

Hypothesis 3: Because of the limited power of tests for FSD (and possibly SSD) it is expected that these tests will be unable to discriminate between the return distributions for NZO and less-diversified portfolios in comparison to more-diversified counterparts.

The theoretical superiority of SD tests in comparison to MV analysis is emphasized in several studies such as Porter et al. (1973) and Kuosmanen (2001). A question that naturally arises is how SD and MV analysis compare regarding portfolio diversification benefits in a head-to-head assessment. Hypothesis 4 is developed to suggest that SD and MV analysis will yield the same (or similar) conclusions.

Hypothesis 4: Evidence of portfolio diversification benefits (internationally-diversified vs. NZO, or more internationally-diversified vs. less internationally-diversified) developed using SD tests is expected to be supported by MV tests of portfolio diversification.

4. Datasets employed, portfolio formation, and methods of analysis

4.1. Datasets utilized and formation of portfolios

Portfolios from a New Zealand investor's perspective is formed. The data on New Zealand[1] and other unit trusts[2] is obtained from Datastream. Six NZ unit trusts have complete monthly prices dating back as far as May 1992 and this month is chosen as the starting point. A six-asset New Zealand only portfolio (P6) serves as the basic comparison portfolio. The least internationally-diversified portfolio is a 10-asset grouping containing four of the original NZ UTs, plus three Australian- and three Hong Kong-based unit trusts. It is termed the NZHA portfolio (P10). Six Japanese-based UTs are added to the NZHA portfolio labeled the NZHAJ (P16A) portfolio. An alternate 16-asset[3] portfolio is formed using the P10 unit trusts plus six U.K. trusts termed the NZHAU (P16B) portfolio. Finally, the most internationally-diversified portfolio uses a combination of the P10 assets, three Japanese and three U.K. unit trusts. This last portfolio is the NZHAJU (P16C) grouping.

The portfolio return inputs utilized in determining the efficient weighting schemes in the ex-ante period are calculated as the arithmetically annualized monthly returns. All non-NZ denominated unit trust prices are first converted into NZ dollars.[4] Mean-variance efficient portfolios are constructed assuming no risk-free asset as the average rates on the NZ government stock (short-term T-Bill equivalent) used in both ex-post periods are higher than the majority of returns generated by the portfolio weighting schemes. A second relevant assumption precludes short sales, because short-selling securities is legally prohibited in New Zealand. A final issue is that minimum and maximum investment weights[5] are explicitly incorporated. Elton and Gruber (1995, pg. 105) note that portfolio managers often are constrained by not making too large (or too small) an investment in one particular asset given the potential risk (costs) involved. Further, evidence in Jagannathan and Ma (2002) supports the concept that imposing portfolio weight constraints reduces the risk in optimally estimated portfolios.

Portfolio variance is calculated using the traditional Markowitz "full-covariance" formula. Points along the efficient frontier are traced out by minimizing risk subject to a given level of return. Further, the weights must be positive and conform to the minima and maxima cited previously. For each of the five portfolios, the PWS that yields the minimum variance portfolio (MVP) serves as the starting point, that is, the first usable set of weights. Working upwards along the frontier, weights are determined for the PWS that produces "target returns" (or "target standard deviations") in 1% (2%) intervals. This procedure produces a maximum of 25 target return PWSs ranging from a return of 8% up to 32%. An identical approach yields a maximum of 28 target standard deviation (StD) PWS ranging from a 12% StD up to deviations of 66%. Because the MVP portfolio effectively targets the lowest variance in the within-portfolio analysis that follows, the MVP weighting scheme is treated as a target StD PWS. This procedure is followed for all five portfolios using ex-ante period data.

The weights determined in the ex-ante period once again are purposefully employed in the ex-post periods. One point concerning the portfolio returns in the two ex-post periods needs to be elaborated. Rather than using an arithmetic mean of the monthly return(s) as in the ex-ante period, the ex-post returns are based on a beginning-of-period to end-of-period buy-and-hold return. This ex-post, buy-and-hold return is meant to represent the achievable return an investor would have earned in the two periods, assuming that they did not rebalance portfolio weights from ex-ante period estimates.[6]

4.2. Stochastic dominance tests and mean variance analysis

4.2.1 Procedure for conducting stochastic dominance tests

Three SD rules are used in this paper to analyze empirical data: first-, second- and third-degree stochastic dominance. A brief outline of the algorithms used to assess stochastic dominance in the framework of uniform discrete distributions is provided here. For a complete, rigorous discussion of these SD criteria, see Levy (1998).

For FSD, n observations of rates of return on portfolios F and G are denoted by x and y, respectively, and ranked in an ascending order as $x_1 \leq x_2 \leq \ldots \leq x_n$ and $y_1 \leq y_2 \leq \ldots \leq y_n$. Each observation x_i (y_i) is assigned an equal probability 1/n. Portfolio F is said to dominate portfolio G by FSD, if and only if $x_i \geq y_i$ for all $i = 1, 2, \ldots, n$ where at least one strict inequality holds.

Now consider SSD. Based on the ranking of n observations and assigning a probability of 1/n to each observation in the FSD case above, define $x_i' = \sum_{j=1}^{i} x_j (y_i' = \sum_{j=1}^{i} y_j)$. Portfolio

F is then said to dominate portfolio G by SSD, if and only if $x'_i \geq y'_i$ for all i = 1, 2, . . . , n where at least one strict inequality holds.

For third-degree stochastic dominance (TSD), the results of x'_i and y'_i calculated in the SSD case above are used and portfolio F is said to dominate portfolio G by TSD if and only if the following three conditions are all met at the same time: (1) $x'_1 > y'_1$ or $x'_L > y'_L$ if $x'_k = y'_k$ for all $1 \leq k < L$ (the left-tail necessary condition for TSD dominance); (2) $x'_n \geq y'_n$ (the mean necessary condition for TSD dominance); and (3) $Q_F^{**}(P) \geq Q_G^{**}(P)$ for all suspect intersection points P. Here, Q_F^{**} and $Q_G^{**}(P)$ are calculated as:

$$Q_F^{**}(P) = \frac{1}{n^2}\left(\sum_{j=1}^{i-1} x'_j + \frac{1}{2}x'_i\right) + \frac{1}{2n}\left(P - \frac{i}{n}\right)(nx^* + x'_i) \text{ and} \tag{1}$$

$$Q_G^{**}(P) = \frac{1}{n^2}\left(\sum_{j=1}^{i-1} y'_j + \frac{1}{2}y'_i\right) + \frac{1}{2n}\left(P - \frac{i}{n}\right)(ny^* + y'_i) \tag{2}$$

$$\text{with } P = \frac{\frac{1}{n}(x'_i - y'_i) + \frac{i}{n}(y_{i+1} - x_{i+1})}{y_{i+1} - x_{i+1}} \text{ and } x^* (= y^*) = (x'_{i+1} - x'_i)\left(P - \frac{i}{n}\right) + \frac{1}{n}x'_i.$$

The violation of any of these three conditions means that F cannot dominate G, and so there is no point in proceeding to see whether the other two conditions hold. It is important to note that FSD dominance implies SSD dominance and SSD dominance implies TSD dominance, but not the reverse. Note also that, if F cannot dominate G, this does not automatically imply the dominance of G over F; it is necessary to check whether G dominates F by going through the above-mentioned procedures. In addition, to reduce the number of comparisons, the transitive rule may be employed.

4.2.2. Mean-variance measures

The issue of the appropriateness of using the Sharpe ratio to assess MV portfolio diversification benefits has been raised earlier. Admittedly, other (effectively) mean-variance measures like the Treynor, Jensen (1968) or information ratio could have been considered. However, these measures are inappropriate (or redundant) in this research for the following reasons. First, as Goodwin (1998) notes the information ratio equals the Sharpe ratio when the benchmark portfolio is the risk-free asset. Second, the choice of an alternate benchmark portfolio is a rather thorny issue. A case might be made that since the reference investor is from New Zealand, then perhaps the NZSE 40 Index would be appropriate. But, trying to estimate betas to calculate the Treynor and Jensen ratios where large proportions of the internationally-diversified portfolios are Japanese- or UK-based unit trusts using the NZSE index does not seem likely to yield reliable beta risk measures. Further, using a composite market index employing a weighted mixture of the major indices from each country of trust origin is also less than ideal. Indeed, there is evidence in Meyer and Rose (2003) that internationally diversified portfolios formed using the weights derived from the single-index, or multiple-index models yield less well-diversified portfolios. Third, as noted in Reilly and Brown (2003, pp. 1111–1116), the Treynor and Jensen measures effectively assume a

Ex-Ante Target Return	Ex-Post Period A $\bar{R}_{pi}$	$\sigma(R_{pi})$	Sharpe Ratio (S_i)	Sharpe Rank	Coefficient of Variation	CV Rank
8%	6.302%	4.200%	-0.6336	25	0.6666	1
9%	6.188%	4.794%	-0.5789	24	0.7747	2
10%	6.082%	5.373%	-0.5362	23	0.8833	3
11%	5.972%	5.998%	-0.4986	22	1.0044	4
12%	5.862%	6.635%	-0.4674	21	1.1317	5
13%	5.759%	7.242%	-0.4423	20	1.2575	6
14%	5.650%	7.892%	-0.4198	19	1.3970	7
15%	5.532%	8.584%	-0.3998	18	1.5518	8
16%	5.420%	9.242%	-0.3833	17	1.7052	9
17%	5.309%	9.904%	-0.3690	16	1.8656	10
18%	5.198%	10.568%	-0.3563	15	2.0332	11
19%	5.080%	11.270%	-0.3445	14	2.2184	12
20%	4.969%	11.938%	-0.3345	13	2.4022	13
21%	4.859%	12.606%	-0.3256	12	2.5946	14
22%	4.748%	13.276%	-0.3175	11	2.7962	15
23%	4.631%	13.983%	-0.3098	10	3.0196	16
24%	4.520%	14.655%	-0.3032	9	3.2421	17
25%	4.410%	15.327%	-0.2971	8	3.4758	18
26%	4.298%	16.037%	-0.2909	7	3.7312	19
27%	4.182%	16.710%	-0.2861	6	3.9955	20
28%	4.072%	17.384%	-0.2814	3	4.2693	21
29%	3.955%	18.095%	-0.2768	2	4.5752	22
30%	3.845%	18.770%	-0.2727	1	4.8822	23
31%	2.793%	21.892%	-0.2818	4	7.8382	24
32%	1.706%	25.568%	-0.2838	5	14.9871	25

Fig. 1. Portfolio 6 Sharpe ratio example.

completely diversified portfolio, and thereby tell nothing about diversification benefits. However, the Sharpe ratio evaluates both return performance and diversification benefits. Finally, there is evidence in Bryant and Eleswarapu (1997) that a CAPM-based risk measure, like beta, is not useful in explaining cross-sectional returns in the New Zealand market.

Evidence suggesting logical problems with the Sharpe ratio is portrayed in Fig. 1. The sample statistics are drawn by employing actual results for Portfolio 6 (NZO) in the ex-post A period. The first column shows the ex-ante period portfolio Target Return for a given PWS. Columns two and three show the portfolio mean return and SD that the (ex-ante period) PWS generates during ex-post A period. For example, the portfolio weighting scheme that yields a target return of 8% in the ex-ante period, when employed in the following subsequent investment interval produces a return of 6.302% and SD of 4.20%. Columns four and five show the Sharpe ratio and its ranking (least negative gets highest ranking). Column six shows the coefficient of variation. Column seven shows coefficients of variation rankings. The portfolio with the CV closest to zero ranks highest.

Fig. 1 shows that the PWS yielding both the highest mean return and lowest SD is for the

8% Target Return. This combination yields a coefficient of variation of 0.666, which ranks as number one in comparison to the CV results for the other 24 PWS shown. To calculate the Sharpe ratio in the ex-post A period, the risk-free rate utilized equals the mean of the NZ 90-day government stock mid-rates over this period, which is 8.963%. Somewhat surprisingly, the 30% Target Return (TR) PWS with a mean return of 3.845% and SD of 18.770% yields the Sharpe ratio that is ranked number one. In comparison to the 8% Target Return portfolio, this 30% TR PWS has a return that is both lower and a SD that is higher. Indeed, Fig. 1 shows that 24 PWS with both a lower mean return and higher SD are ranked higher than that of the 8% TR portfolio on the basis of the Sharpe ratio. In contrast, the simpler CV ranks the 30% TR portfolio in 23rd place. When viewed as a whole, Fig. 1 calls into question the logic of the Sharpe ratio rankings.

5. Empirical results

5.1. Stochastic dominance tests for diversification benefits across portfolios

Table 1 shows the stochastic dominance test summary results between New Zealand-only and internationally-diversified unit trust portfolios based on the target return weighting schemes. The table is split into four panels based on the ex-ante, two ex-post and combined ex-post periods. The columns are organized on the basis of the degree of SD test shown. Perhaps the most striking result in Table 1 is shown in columns four and five and offers evidence on Hypothesis 3. Specifically, using first-degree SD tests, pair-wise comparisons of the five portfolios show that neither portfolio exhibits FSD in any comparison, in any period. The results depicted in columns six and seven for second-order SD show that SSD tests are significant in roughly one-half (22/40) of all comparisons. But the third-order SD tests show the greatest evidence of stochastic dominance, as 39 of 40 comparisons are significant. Because the SSD and TSD dominance results are internally consistent with each other, discussion of the pair-wise analysis will focus largely on the TSD findings. From an investor or analyst's perspective these results show that conducting first-order SD tests is essentially useless.

The null version of Hypothesis 1 is that adding (more) international unit trusts to a wholly domestic portfolio will yield increased diversification benefits. The TSD (and also SSD) tests do not support Hypothesis 1, independent of the period of analysis. In comparing P6 to the more-diversified portfolios the NZO portfolio is found to be stochastically dominant in 16 of 16 cases across all four periods. Further, P10 similarly dominates its more-diversified counterparts in 10 of 10 cases, again across all periods. The only limited evidence of benefits through increased international diversification shown in Table 1 is because of P16C, the most diversified portfolio, but only as compared to its 16-asset counterparts. Altogether, this evidence does not support the benefits of increased diversification and is in direct contrast to the MV evidence of international diversification benefits shown in Jorion (1985), Grauer and Hakkansson (1987), and Meyer and Rose (2003). Table 1 also provides evidence supporting Hypothesis 2 that portfolio SD is expected to persist across time frames. Portfolios P6 and P10, which stochastically dominate in the ex-ante period, are similarly found to dominate in

Table 1
Stochastic dominance paired sample comparison for all target return combinations

Period	Comparison		FSD Portfolio dominates	Freq	SSD Portfolio dominates	Freq	TSD Portfolio dominates	Freq
Ex-ante (May 1992-Jan 1995)	P6 vs.	P10	Neither	0/20	P6	11/20	P6	20/20
		P16A	Neither	0/24	Neither	0/24	P6	24/24
		P16B	Neither	0/24	P6	4/24	P6	24/24
		P16C	Neither	0/21	P6	1/21	P6	21/21
	P10 vs.	P16A	Neither	0/20	Neither	0/20	P10	19/20
		P16B	Neither	0/20	Neither	0/20	P10	19/20
		P16C	Neither	0/20	Neither	0/20	P10	17/20
	P16A vs.	P16B	Neither	0/24	P16A	12/24	P16A	16/24
		P16C	Neither	0/21	Neither	0/21	Neither	0/21
	P16B vs.	P16C	Neither	0/21	P16C	9/21	P16C	15/21
Ex-post A (Feb 1995-May 1996)	P6 vs.	P10	Neither	0/20	P6	20/20	P6	20/20
		P16A	Neither	0/24	P6	24/24	P6	24/24
		P16B	Neither	0/24	Neither	0/24	P6	24/24
		P16C	Neither	0/21	P6	21/21	P6	21/21
	P10 vs.	P16A	Neither	0/20	P10	20/20	P10	20/20
		P16B	Neither	0/20	Neither	0/20	P10	20/20
		P16C	Neither	0/20	P10	20/20	P10	20/20
	P16A vs.	P16B	Neither	0/24	P16B	24/24	P16B	24/24
		P16C	Neither	0/21	P16C	21/21	P16C	21/21
	P16B vs.	P16C	Neither	0/21	P16B	20/21	P16B	20/21[a]
Ex-post B (June 1996-May 1998)	P6 vs.	P10	Neither	0/20	P6	20/20	P6	20/20
		P16A	Neither	0/24	P6	22/24	P6	22/24[b]
		P16B	Neither	0/24	Neither	0/24	P6	24/24
		P16C	Neither	0/21	Neither	0/21	P6	21/21
	P10 vs.	P16A	Neither	0/20	P10	14/20	P10	17/20
		P16B	Neither	0/20	Neither	0/20	P10	20/20
		P16C	Neither	0/20	P16C	3/20	P10	16/20[c]
	P16A vs.	P16B	Neither	0/24	Neither	0/24	P16A	24/24
		P16C	Neither	0/21	P16C	18/21	P16C	18/21[d]
	P16B vs.	P16C	Neither	0/21	Neither	0/21	P16C	21/21
Combined Ex-post pds. (Feb 1995-May 1998)	P6 vs.	P10	Neither	0/20	P6	20/20	P6	20/20
		P16A	Neither	0/24	Neither	0/24	P6	24/24
		P16B	Neither	0/24	Neither	0/24	P6	24/24
		P16C	Neither	0/21	Neither	0/21	P6	21/21
	P10 vs.	P16A	Neither	0/20	P10	10/20	P10	20/20
		P16B	Neither	0/20	Neither	0/20	P10	20/20
		P16C	Neither	0/20	Neither	0/20	P10	20/20
	P16A vs.	P16B	Neither	0/24	P16B	8/24	P16A	16/24[e]
		P16C	Neither	0/21	P16C	18/21	P16C	18/21[f]
	P16B vs.	P16C	Neither	0/21	Neither	0/21	P16C	20/21

[a] One 16C portfolio dominates. [b] Two P16A portfolios dominate. [c] Three P16C portfolios prove dominant. [d] One P16A portfolio dominates. [e] Eight P16B portfolios dominate. [f] Three P16A portfolios dominate.

P6 is the six-asset New-Zealand Only (NZO) portfolio. P10 (NZHA) is a 10-asset portfolio composed of New Zealand (4), Australian (3), and Hong Kong (3) unit trusts. P16A (NZHAJ) is a 16-asset portfolio that contains the 10 NZHA assets plus six Japanese unit trusts. P16B (NZHAU) is a 16-asset portfolio containing the NZHA assets plus six unit trusts from the United Kingdom. P16C (NZHAJU) is a 16-asset portfolio formed with the NZHA assets plus three Japanese and three UK unit trusts. FSD, SSD, or TSD refers to first-, second-, and third-order stochastic dominance, respectively. Freq (frequency) reports the ratio of portfolio weighting schemes (in a given portfolio) that stochastically dominate their paired counterpart.

both subsequent periods, as well as over the combined time frame of analysis. This evidence of persistent dominance should reassure investors regarding performance characteristics.

The SD test results for international diversification benefits between the five portfolio combinations, across the four periods, are repeated for the portfolios formed using target StD weighting schemes. These results are reported in Table 2 and they largely mirror the tests based on target return weighting schemes. Briefly, FSD proves to have no discriminatory power, less diversified portfolios largely exhibit SD over their more diversified counterparts and evidence of SD from period-to-period is persistent. Given that the PWS in Table 2 are developed on the basis of target risk rather than return, the constancy of results is evidence of robustness.

5.2. Stochastic dominance tests within portfolios

The SD tests also address the question of whether a particular portfolio-weighting-scheme-focus stochastically dominates others within a given portfolio. A second issue is again whether any dominance is persistent beyond the formation period. The results of this analysis are shown in Table 3. Again, FSD tests are found to offer no significant discriminatory ability between PWS within the portfolios, in any period. Further, Table 3 shows that in the ex-ante period, no particular portfolio focus (equivalently, no PWS), whether high, medium, or low return or risk, is stochastically dominant. The finding that the schemes specifically developed to yield the highest return is not dominant is quite interesting given that perfect foresight is effectively applied in the ex-ante period to determine efficient portfolios. Table 3 also provides evidence regarding stochastically dominant portfolio weighting schemes within portfolios in the two out-of-sample ex-post periods. For the P6 portfolio, the lowest target return portfolio (R08) and the lowest target SD portfolio (S12) have the highest mean returns in the ex-post A and ex-post B periods, as well as the combined period. Based on both SSD and TSD tests, this PWS dominates all other weighting schemes. The results for portfolio P10 in regard to SD portfolio weighting schemes virtually mirror those described above for P6. This result is consistent across both periods and supports Hypothesis 2.

The Table 3 results suggest two outcomes. First, in contrast to the non-dominant high return or high risk target PWSs in the ex ante period, the lowest return/lowest risk weighting schemes (18 of 20) produce the highest mean returns (and/or exhibit pervasive dominance) in the ex-post periods. Second, this evidence of stochastic dominance applies to both ex-post periods and is evidence of persistence, supporting Hypothesis 2. The analysis from Table 3 supports a central tenet of finance theory, that is, that there is a direct relationship between risk and return. In the ex-ante period the NZ, Japanese and Hong Kong markets exhibit the best relative performances. The efficient weighting schemes developed tended to give some of the UTs (especially the high return/high risk assets) in these markets larger weights. However, in the two ex-post periods these high-risk assets suffered disproportionately from market conditions directly before, and during the Asian Crisis. Thus, in the ex-post periods, the PWS originally developed to produce low (target) returns or low (target StD) risk suffer less from the economic downturn.

Table 2
Stochastic dominance paired sample comparison for all target SD combinations

Period	Comparison		FSD Portfolio dominates	Freq	SSD Portfolio dominates	Freq	TSD Portfolio dominates	Freq
Ex-ante (May 1992-Jan 1995)	P6 vs.	P10	Neither	0/27	P6	17/27	P6	27/27
		P16A	Neither	0/28	Neither	0/28	P6	28/28
		P16B	Neither	0/28	P6	5/28	P6	28/28
		P16C	Neither	0/27	Neither	0/27	P6	27/27
	P10 vs.	P16A	Neither	0/27	Neither	0/27	P10	27/27
		P16B	Neither	0/27	P10	2/27	P10	27/27
		P16C	Neither	0/26	Neither	0/26	P10	24/26
	P16A vs.	P16B	Neither	0/28	P16A	16/26	P16A	19/26
		P16C	Neither	0/27	P16A	1/27	P16A	1/27
	P16B vs.	P16C	Neither	0/27	P16C	11/27	P16C	19/27
Ex-post A (Feb 1995-May 1996)	P6 vs.	P10	Neither	0/27	P6	27/27	P6	27/27
		P16A	Neither	0/28	P6	28/28	P6	28/28
		P16B	Neither	0/28	P6	5/28	P6	28/28
		P16C	Neither	0/27	P6	26/27	P6	27/27
	P10 vs.	P16A	Neither	0/27	P10	27/27	P10	27/27
		P16B	Neither	0/27	Neither	0/27	P10	0/27
		P16C	Neither	0/26	P10	24/26	P10	26/26
	P16A vs.	P16B	Neither	0/28	Neither	0/28	P16B	28/28
		P16C	Neither	0/27	P16C	25/27	P16C	25/27[a]
	P16B vs.	P16C	Neither	0/27	P16B	3/27	P16B	24/27[b]
Ex-post B (June 1996-May 1998)	P6 vs.	P10	Neither	0/27	P6	24/27	P6	27/27
		P16A	Neither	0/28	P6	28/28	P6	28/28
		P16B	Neither	0/28	Neither	0/28	P6	28/28
		P16C	Neither	0/27	Neither	0/27	P6	27/27
	P10 vs.	P16A	Neither	0/27	P10	22/27	P10	24/27
		P16B	Neither	0/27	Neither	0/27	P10	27/27
		P16C	Neither	0/26	Neither	0/26	P10	24/26
	P16A vs.	P16B	Neither	0/28	Neither	0/28	P16A	28/28
		P16C	Neither	0/27	P16C	22/27	P16C	22/27[c]
	P16B vs.	P16C	Neither	0/27	Neither	0/27	P16C	27/27
Combined ex-post pds. (Feb 1995-May 1998)	P6 vs.	P10	Neither	0/27	P6	27/27	P6	27/27
		P16A	Neither	0/28	P6	28/28	P6	28/28
		P16B	Neither	0/28	Neither	0/28	P6	28/28
		P16C	Neither	0/27	Neither	0/27	P6	27/27
	P10 vs.	P16A	Neither	0/27	P10	13/27	P10	27/27
		P16B	Neither	0/27	Neither	0/27	P10	27/27
		P16C	Neither	0/26	Neither	0/26	P10	26/26
	P16A vs.	P16B	Neither	0/28	Neither	0/28	P16A	18/28[d]
		P16C	Neither	0/27	P16C	22/27	P16C	22/27[e]
	P16B vs.	P16C	Neither	0/27	P16B	4/27	P16C	22/27[f]

[a] One 16A portfolio dominates. [b] Three P16C portfolios dominate. [c] Five P16A portfolios dominate. [d] Ten P16B portfolios dominate. [e] Four P16A portfolios dominate. [f] Five P16B portfolios dominate their P16C counterparts.

P6 is the six-asset New-Zealand Only (NZO) portfolio. P10 (NZHA) is a 10-asset portfolio composed of New Zealand (4), Australian (3), and Hong Kong (3) unit trusts. P16A (NZHAJ) is a 16-asset portfolio that contains the 10 NZHA assets plus six Japanese unit trusts. P16B (NZHAU) is a 16-asset portfolio containing the NZHA assets plus six unit trusts from the United Kingdom. P16C (NZHAJU) is a 16-asset portfolio formed with the NZHA assets plus three Japanese and three UK unit trusts. FSD, SSD or TSD refers to first-, second-, and third-order stochastic dominance, respectively. Freq (frequency) reports the ratio of portfolio weighting schemes (in a given portfolio) that stochastically dominate their paired counterpart.

Table 3
Stochastic dominance results for comparisons within portfolios based on targeted return and risk weighting schemes

Portfolio	Period	StochDom type	Target returns		Target StDs	
			Highest mean	Dominate lower μ's	Highest mean	Dominate lower μ's
P6	Ex-ante	FSD	R32	None	S66	None
	(May 1992-	SSD	R32	None	S66	None
	Jan 1995)	TSD	R32	None	S66	None
	Ex-postA	FSD	R08	None	S12	None
	(Feb 1995-	SSD	R08	All	S12	All
	May 1996)	TSD	R08	All	S12	All
	Ex-postB	FSD	R08	Yes (1/25)	S12	Yes (1/28)
	(June 1996-	SSD	R08	All	S12	All
	May 1998)	TSD	R08	All	S12	All
	Combined	FSD	R08	None	S12	None
	(Feb 1995-	SSD	R08	All	S12	All
	May 1998)	TSD	R08	All	S12	All
P10	Ex-ante	FSD	R27	None	S66	None
	(May 1992-	SSD	R27	None	S66	None
	Jan 1995)	TSD	R27	None	S66	None
	Ex-postA	FSD	R08	None	S14	None
	(Feb 1995-	SSD	R08	All	S14	Yes (24/27)
	May 1996)	TSD	R08	All	S14	All
	Ex-postB	FSD	R08	None	S14	None
	(June 1996-	SSD	R08	All	S14	All
	May 1998)	TSD	R08	All	S14	All
	Combined	FSD	R08	None	S14	None
	(Feb 1995-	SSD	R08	All	S14	All
	May 1998)	TSD	R08	All	S14	All
P16A	Ex-ante	FSD	R31	None	S66	None
	(May 1992-	SSD	R31	None	S66	None
	Jan 1995)	TSD	R31	None	S66	None
	Ex-postA	FSD	R08	None	S12	None
	(Feb 1995-	SSD	R08	Yes (23/24)	S12	All
	May 1996)	TSD	R08	Yes (23/24)	S12	All
	Ex-postB	FSD	R17	None	S30	None
	(June 1996-	SSD	R17	Yes (14/24)	S30	Yes (18/28)
	May 1998)	TSD	R17	Yes (14/24)	S30	Yes (18/28)
	Combined	FSD	R08	None	S12	None
	(Feb 1995 to	SSD	R08	All	S12	All
	May 1998)	TSD	R08	All	S12	All
P16B	Ex-ante	FSD	R30	None	S62	None
	(May 1992-	SSD	R30	None	S62	None
	Jan 1995)	TSD	R30	None	S62	None
	Ex-postA	FSD	R31	None	SMVP	None
	(Feb 1995-	SSD	R31	Yes (4/24)	SMVP	Yes (20/28)
	May 1996)	TSD	R31	Yes (4/24)	SMVP	Yes (20/28)
	Ex-postB	FSD	R08	None	SMVP	None
	(June 1996-	SSD	R08	Yes (19/24)	SMVP	Yes (22/28)
	May 1998)	TSD	R08	Yes (19/24)	SMVP	Yes (22/28)
	Combined	FSD	R09	None	SMVP	None
	(Feb 1995-	SSD	R09	Yes (19/24)	SMVP	Yes (22/28)
	May 1998)	TSD	R09	Yes (19/24)	SMVP	Yes (22/28)

(*continued on next page*)

Table 3
Continued

Portfolio	Period	StochDom type	Target returns: Highest mean	Target returns: Dominate lower μ's	Target StDs: Highest mean	Target StDs: Dominate lower μ's
P16C	Ex-ante	FSD	R28	None	S64	None
	(May 1992-	SSD	R28	None	S64	None
	Jan 1995)	TSD	R28	None	S64	None
	Ex-postA	FSD	R08	None	SMVP	None
	(Feb 1995-	SSD	R08	All	SMVP	All
	May 1996)	TSD	R08	All	SMVP	All
	Ex-postB	FSD	R08	None	SMVP	None
	(June 1996-	SSD	R08	All	SMVP	Yes (2/27)
	May 1998)	TSD	R08	All	SMVP	Yes (2/27)
	Combined	FSD	R08	None	SMVP	None
	(Feb 1995-	SSD	R08	All	SMVP	All
	May 1998)	TSD	R08	All	SMVP	All

P6 is the six-asset New-Zealand Only (NZO) portfolio. P10 (NZAH) is a 10-asset portfolio composed of New Zealand (4), Australian (3), and Hong Kong (3) unit trusts. P16A (NZAHJ) is a 16-asset portfolio that contains the ten NZAH assets plus six Japanese unit trusts. P16B (NZAHU) is a 16-asset portfolio containing the NZAH assets plus six unit trusts from the United Kingdom. P16C (NZAHJU) is a 16-asset portfolio formed with the NZAH assets plus three Japanese and three U.K. unit trusts. FSD, SSD, or TSD refers to first-, second-, and third-order stochastic dominance (StochDom), respectively. In the "Highest mean" columns, Rxx (Sxx) refers to the portfolio weighting scheme that produces a return (standard deviation) of xx percentage in the ex-ante portfolio formation period. In the "Dominate lower μ's" (means) column, the ratio in parentheses denotes how many comparisons, out of the total, yield a significant dominating result. SMVP denotes the target standard deviation minimum variance portfolio.

5.3. Mean-variance analysis of portfolio diversification benefits

The SD evidence supports the idea that for the portfolios and time periods analyzed, less well-diversified portfolios exhibit dominance. This is evidence against the null version of Hypothesis 1 predicting benefits from greater diversification. A question of interest is whether the same conclusion would be reached on the basis of mean-variance analysis. Hypothesis 4 is specifically developed to test if the SD findings in regard to international diversification benefits are supported by MV analysis.

Table 4 provides the results of MV analysis. As the previous target return and target SD portfolio weighting schemes produce essentially the same results, in this analysis all PWS within a given portfolio are aggregated to conduct tests for international diversification benefits across portfolios. The four panels in Table 4 depict the portfolio means, SD and *t*-tests for the ex-ante, two ex-post periods, and the combined ex-post periods. Parametric *t*-tests for significant differences in means using two MV measures are shown in Table 4. The first measure of the return-risk tradeoff for the MV analysis is the Sharpe ratio. Given the concerns with the Sharpe ratio described above, the coefficient of variation is also employed in the MV analysis. In the discussion below that cites comparisons based on a particular measure of performance, there are technically a total of 20 comparisons, that is, five portfolios versus four other portfolios. However, since the description starts with P6 versus

Table 4
t-Tests for significant differences between Sharpe ratios and coefficients of variation across portfolios

Period	Portfolio	Mean $\mu(R_p)$ (%)	$SD(R_p)$ (%)	Sharpe	Coef Var	P6 vs. Sharpe *t*-Tst	P6 vs. CoefVar t-Tst	P10 vs. Sharpe *t*-Tst	P10 vs. CoefVar *t*-Tst	P16A vs. Sharpe *t*-Tst	P16A vs. CoefVar *t*-Tst	P16B vs. Sharpe *t*-Tst	P16B vs. CoefVar *t*-Tst
Ex-ante	P6	18.41	42.13	0.2569	2.1989	—	—	—	—	—	—	—	—
(May 1992-	P10	16.98	39.96	0.2329	2.3146	0.1102	0.0043	—	—	—	—	—	—
Jan 1995)	P16A	19.51	37.98	0.3147	1.8814	0.0001	0.0001*	0.0001*	0.0001*	—	—	—	—
	P16B	19.18	38.59	0.2950	1.9636	0.0145	0.0000	0.0005	0.0001*	0.2538	0.0133	—	—
	P16C	18.78	35.33	0.3240	1.7972	0.0001*	0.0001*	0.0001*	0.0001*	0.5784	0.0675	0.0934	0.0001
Ex-post A	P6	5.08	10.81	−0.4042	2.4750	—	—	—	—	—	—	—	—
(Feb 1995 to	P10	5.88	12.79	−0.2395	2.2302	0.0001*	0.4248	—	—	—	—	—	—
May 1996)	P16A	2.63	19.49	−0.3365	10.0557	0.0004	0.0001*	0.0001*	0.0001*	—	—	—	—
	P16B	8.85	20.72	−0.0164	2.3045	0.0001*	0.5873	0.0001*	0.5830	0.0001*	0.0001*	—	—
	P16C	3.94	21.41	−0.2373	6.1664	0.0001*	0.0001*	0.6454	0.0001*	0.0001*	0.0001*	0.0001*	0.0001*
Ex-post B	P6	−2.46	16.85	−0.6778	−4.5385	—	—	—	—	—	—	—	—
(June 1996-	P10	−3.01	23.15	−0.4833	−3.8698	0.0001*	0.8780	—	—	—	—	—	—
May 1998)	P16A	−4.62	22.77	−0.5571	−14.1439	0.0001*	0.2575	0.0001*	0.2465	—	—	—	—
	P16B	−3.24	28.97	−0.3747	−5.8850	0.0001*	0.7973	0.0001*	0.7303	0.0001*	0.3751	—	—
	P16C	−1.68	23.43	−0.4118	−24.0808	0.0001*	0.4008	0.0001*	0.3879	0.0001*	0.6840	0.0357	0.4400
Combined	P6	7.01	23.27	−0.2750	0.0451	—	—	—	—	—	—	—	—
(May 1992 to	P10	6.62	25.30	−0.1633	0.2250	0.0063	0.9036	—	—	—	—	—	—
May 1998)	P16A	5.84	26.75	−0.1930	−0.7356	0.0620	0.7895	0.4518	0.7524	—	—	—	—
	P16B	8.26	29.43	−0.0320	−0.5390	0.0004	0.7429	0.0001	0.6983	0.0001*	0.9510	—	—
	P16C	7.01	26.72	−0.1084	−5.3724	0.0001	0.4839	0.1362	0.4721	0.0362	0.5707	0.0295	0.5378

* Indicates *p*-value is actually closer to zero than indicated.

P6 is the six-asset New-Zealand Only (NZO) portfolio. P10 (NZHA) is a 10-asset portfolio composed of New Zealand (4), Australian (3), and Hong Kong (3) unit trusts (UTs). P16A (NZHAJ) is a 16-asset portfolio that contains the ten NZHA assets plus six Japanese UTs. P16B (NZAHU) is a 16-asset portfolio containing the NZAH assets plus six UTs from the U.K. P16C (NZAHJU) is a 16-asset portfolio formed with the NZHA assets plus three Japanese and three U.K. UTs. $\mu(R_p)$ is the mean return across all portfolio-weighting schemes within a given portfolio, over the period indicated, while $SD(R_p)$ is the StD of the portfolio returns. Sharpe (Coef Var) is the Sharpe ratio (coefficient of variation) calculated on the basis of the mean and StD shown in the table. Sharpe (CoefVar) *t*-Test is actually the *t*-Test *p*-value testing for a significant difference between the Sharpe ratio (CoefVar) for portfolio indicated and the other portfolios indicated. The *p*-value is the probability of observing a more extreme, sample *t*-statistic.

the other four portfolios, the comparative discussion of P10, for example, will only relate to the remaining three portfolios that have not already been discussed, and so on.

The top panels of ex-ante period results are in marked contrast to the stochastic dominance results. Of the 20 combined across-portfolio tests (for both Sharpe ratios and CV), 15 are significant, at a minimally 5% level, based on the *p*-values shown. In fact, the significant results are almost directly opposite to those found using the SD tests. For example, seven of the eight P6 tests are significant, and of these seven, six are interpretable as showing that the more diversified portfolio has a significantly higher mean Sharpe ratio or a CV that is closer to zero. For P10, this conclusion is even stronger given that all six comparisons (excluding those P6 ones previously discussed) show that the more diversified portfolios have significantly better MV measures than their less diversified counterparts. Of the six remaining comparisons, three are significant, and two support Hypothesis 1 regarding increased benefits because of international diversification.

In the ex-post A period, the P6 portfolio has a significantly poorer Sharpe ratio compared to all four other portfolios. Two of the three P10 Sharpe ratio comparisons are significant. Both 16B and P16C offer a significantly better tradeoff than P16A, while P16C is significantly better than P16B. In sum, more diversified portfolios are superior in seven of nine cases. The Sharpe ratio results in ex-post period B are nearly identical to those from ex-post period A. Considering the significant finding that P16B return-risk tradeoff is preferred to P16C, the overall score supporting greater diversification benefits for the Sharpe ratios in this period, is eight out of 10. The final period of analysis depicted in Table 4 is for the two ex-post periods combined. Interestingly, the Sharpe ratio statistics exhibit benefits from increased diversification in only 50% (5/10) of the cases here. The Sharpe ratio results suggest more diversified portfolios generally offer a better risk-return tradeoff (supporting Hypothesis 1) and that MV-test, and SD-test results do not support each other (rejecting Hypothesis 4).

Conversely, the coefficient of variation test results are generally consistent with the SD-test findings. Two of the four P6 CV comparisons are significant in the ex-post A period, but they both show the P6 portfolio to be superior. Two of the three P10 CV comparisons are significant and both show P10 to be better. Overall, of the seven significant CV tests, five actually show that less diversification is preferred. None of the CV t-tests turn out to be significant in ex-post period B. In the combined (ex-post) period, the consistent insignificance of the CV comparisons mirrors the results from ex-post period B. Thus, less diversified portfolios appear superior to those that are more diversified (rejecting Hypothesis 1). This rejection coincides with the findings from the SD analysis suggesting at least one MV measure complements the SD analysis (supporting Hypothesis 4). A further conclusion is that investors should view analysis conducted solely on the basis of Sharpe ratios with some degree of caution.

6. Summary and conclusions

The objective of this research is to apply tests for stochastic dominance to the question of whether internationally diversified portfolios will dominate a wholly domestic portfolio formed of assets from a small, developed market perspective. A parallel question is whether increasingly more diversified portfolios will dominate their less-diverse counterparts. Both of these questions are important to investors, and portfolio managers wishing to optimize risk adjusted returns.

Stochastic dominance tests compare five portfolios to each other in three non-overlapping periods. The NZ-only portfolio (P6) is found to stochastically dominate the other four portfolios in the ex-ante (May 1992-January 1995), ex-post A (February 1995-May 1996), ex-post B (June 1996-May 1998) and the combined ex-post periods. Further, the second least diversified portfolio (termed P10) similarly proves to dominate its three more diverse counterparts across all four periods. These results do not support the hypothesis that adding international assets to a wholly domestic portfolio generate increased diversification benefits. Nor do they suggest that increased international diversification improves portfolio performance.

The across-portfolio analysis also yields evidence on two other issues. First, the fact that the SD exhibited by P6 and P10 applies across all periods analyzed shows that the evidence regarding diversification benefits is persistent. Second, of the three types of stochastic dominance tests employed, first-order SD tests are unable to distinguish whether any portfolio dominates. More restrictive second-order SD tests prove able to detect dominance in 45 of 80 across-portfolio tests while third-order SD tests prove able to determine dominance in 79 of 80 comparisons.

A second application of SD analysis is to determine if a particular portfolio investment focus dominates another focus persistently. These results indicate when markets experience greater uncertainty or even economic crisis, low risk or low return investment strategies dominate. This fact is interesting as it supports the most basic tenet of finance, that there is a direct relationship between risk and return.

This research also examines the relative comparability of the SD findings with the results of parallel mean-variance analysis. The Sharpe ratio and the coefficient of variation are used as measures of the MV risk-return tradeoff. *t*-tests for differences in portfolio Sharpe ratio means support the hypothesis that more diversified portfolios offer a superior risk-return tradeoff. This finding is opposite to the SD test results in which the least diversified portfolios prove to dominate their more diversified counterparts. However, questions are raised about the logical consistency of the Sharpe ratio in some situations, so the coefficient of variation is also used to analyze diversification benefits. In sum, the CV tests are found to exhibit a closer correspondence to SD-test results.

To conclude, recent advances in stochastic dominance tests make it a usable tool when investors are assessing the benefits of greater portfolio diversification. Given its theoretical superiority over traditional mean-variance analysis, it should also prove a versatile analytical tool in other finance applications. Finally, a general conclusion from this research is that in at least some applications, for example, in conditions of high market volatility, SD analysis could be used in conjunction with mean-variance analysis to add additional insight into portfolio formulation decisions.

Notes

1. Unit trusts are employed in this study, rather than individual stocks, as Pinfold, Wilson, and Li (2001, p.301) conclude that "the small number of companies listed

makes it impossible to construct portfolios that provide a satisfactory degree of diversification."

2. Unit trusts based in Australia, Hong Kong, Japan, and the U.K. are chosen to provide international diversification in this study as these countries are all important trading partners for New Zealand and thus might be thought to represent natural choices for a NZ investor. No U.S. mutual funds are employed as preliminary analysis conducted on a random sample of 100 funds, during the ex-ante period, showed that the U.S. funds' risk-return characteristics tended to dominate the unit trusts chosen for inclusion.
3. Sixteen assets are chosen as the maximum number of UTs in a given portfolio based on research in O'Neal (1997) showing that the reduction of terminal wealth variability via increasing the number of mutual funds in a portfolio (formed of mutual funds) after 16, increases at a decreasing rate.
4. The sensitivity of the potential results to exchange rate effects is analyzed through a comparison of the converted and non-converted returns. These results suggest that exchange-rate conversions have only a minor impact on the international diversification benefits analysis.
5. The minimum weight for the six- and 10-asset portfolios is 5%, while that for the 16-asset portfolios equals 2%. The maximum weights for the six-, 10- and 16-asset portfolios are 60, 55, and 50%, respectively.
6. The return calculations and analysis are also conducted inclusive of transactions costs as represented by buying at the entry (ask) price and selling at the exit (bid) price, as well as including additional commissions characterized by a sample of NZ brokers as typical. The conclusions regarding diversification benefits are not substantively different from those reported in this study.

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