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Hitting or missing the retirement target: comparing contribution and asset allocation schemes of simulated portfolios

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Abstract

Personal financial planning entails establishing retirement portfolio goals, which may be identified as specific portfolio target values. The problem of investing to build a retirement portfolio that achieves a specified value at retirement can be modeled as a dynamic multiperiod portfolio problem. Although theoretically robust, the dynamic programming approach to multiperiod portfolio analysis is computationally intractable. In contrast, this paper uses Monte Carlo simulation to analyze specific asset allocations in the context of multiperiod planning. We consider achieving a target portfolio value at a specific point in time. We conclude that 'life-cycle funds' that reduce equity allocations over time fail to increase the likelihood of reaching a targeted portfolio value compared with fixed asset allocation models (e.g., 80% equity/20% bond). © 2007 Academy of Financial Services. All rights reserved.

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1. Introduction

The majority of the vast literature on portfolio analysis considers the trade-off between risk and return in the context of a single period portfolio problem. Although variance and

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volatility are useful risk measures in the Capital Asset Pricing Model (CAPM), such measures of risk may have limited applicability for individual investors seeking to identify suitable asset allocations for multiperiod planning. Our models apply value at risk (VAR) methods for analyzing risk by examining probabilities of failing to achieve various target portfolio values.

We contrast two different contribution strategies to illustrate the type of risks faced by the investor. First, we assume that investors specify a constant real dollar amount to be invested each year. The expected rates of return for a chosen asset allocation strategy are used to determine the expected value of the portfolio when the investor reaches a specific retirement age; for example, 30 years from time t. One of the risks with this approach is that year-to-year uncertainty of actual portfolio returns may cause the portfolio to fall short of its target. Basing a contribution strategy on expected returns exposes any retirement portfolio to substantial risk of failing to reach targeted values. An investor may choose to increase the scale of annual contributions to reduce the probability of not reaching the targeted value, but extraordinarily high levels of contributions may be necessary to achieve desired risk reduction.

Alternatively, an investor may adjust the contribution amounts so that targeted portfolio balances are achieved each year. This approach guarantees that the targeted portfolio value will be achieved at the time the investor retires. The risk inherent in this dynamic approach is that poor portfolio performance may require substantial increases in contribution amounts in some years to achieve year-to-year targeted portfolio amounts.

The risks associated with the two contribution strategies contrasted above can be summarized as follows. Either an investor faces substantial variation of the value of the portfolio at retirement, or she faces substantial variation in the annual amounts saved and invested. In the former case, specified amounts are invested with no guarantee of reaching the investment target. In the latter case, the target is achieved but reaching the target may cost dearly in terms of annual contribution amounts.

Regardless of contribution strategy, however, overall portfolio performance depends heavily upon asset allocation strategy. Recently financial institutions, especially mutual fund companies, have been promoting ‘life-cycle’ or ‘target’ mutual funds in which the asset allocation shifts as one approaches retirement age (e.g., Laise, 2006). Assets in life-cycle funds increased by 60% in 2005 to $70.1 billion (Ibid). Although specific asset allocations for each life-cycle fund differ, all of these allocation strategies advocate shifts away from equities in favor of bonds and cash as the investor approaches a retirement target date (e.g., Israelsen & Walker, 2006).

The analyses in this paper serve two purposes. First, we compare two investment strategies by observing the risks associated with failing to meet a predetermined retirement portfolio target. For illustrative and simplicity purposes, a target amount has been set at $1,000,000 with a planning period of 30 years. We determine the constant real dollar annual contribution amounts by amortizing the $1,000,000 target using expected real rates of return. We demonstrate that for all of the hypothetical portfolios analyzed, the chances are less than one-half of meeting the one million dollar retirement goal if one employs a strategy of investing a constant real dollar amount annually. Furthermore, there is substantial risk of failing to the meet the one million dollar target, even if one significantly increases the annual amounts invested. In contrast, the strategy that invests the minimum amount each year to achieve a predetermined balance is guaranteed to achieve or exceed the target. We observe, however, that the amount one must invest in certain years may be significantly greater than the mean, so much, so that such a strategy may be impractical for many investors.

Second, we observe the effects of various asset allocation 'life-cycle' strategies on meeting the retirement portfolio target. We compare several generic asset allocation strategies to portfolios that mimic life-cycle fund allocations promoted by financial services companies. Our results, which are similar to other studies such as Vora and McGinnis (2000), indicate that asset allocations should be weighted towards equities, even when few years remain in the non-retirement investment years. This result contradicts the conventional wisdom inherent in most life-cycle allocations that call for shifts away from equities and toward less volatile investment instruments as an investor ages.

2. Literature review

Standard risk measures such as volatility and standard deviation are fundamental to the CAPM. The CAPM, is a single period model whereas the portfolio retirement problem we consider is multiperiod. Elton and Gruber (1975) use dynamic programming to analyze multiperiod portfolio problems that maximize the utility of terminal wealth. The Elton and Gruber work is based on the seminal scholarship of Merton (1969; 1971), Mossin (1968), and Samuelson (1969). These early works provide useful insight about the structure of optimal portfolios. The primary problem with the dynamic programming approach to multiperiod portfolios is computational intractability. The alternative method of Monte Carlo simulation is used by Booth (2004) in analyzing retirement targets and asset allocation. Vora and McGinnis (2000) address optimal asset allocation for retirement portfolios as the retiree draws down the portfolio. Contrary to conventional wisdom, they conclude that retirement portfolios should rely heavily on equities. We reach similar conclusions for the investor who is building a retirement portfolio.

Monte Carlo simulation is applied by Cooley, Hubbard and Walz (2003) to the sustainability of retirement portfolios. In contrast, our paper examines the problem of reaching a portfolio target goal by the time the investor reaches retirement. Farrell (2001), Milevsky and Panyagometh (2001), Savage (2004), and Savage (2006) address a range of portfolio issues and problems of mutual funds using Monte Carlo simulation.

3. Models

For simplicity, consider a 30-year-old investor who seeks to amass \$1,000,000 (current dollars) at age 60. She has various investment options (both in her choice of investment instruments and in the amount she chooses to invest annually) to achieve her desired \$1,000,000 goal.

Table 1 Asset allocations

Fund	Year 1		Year 30		Constant payment	Fund risk
	Large caps	LT corp bonds	Large caps	LT corp bonds		
Life Cycle Fund I	85%	15%	60%	40%	9,730	Aggressive
Life Cycle Fund II	78%	22%	60%	40%	10,194	Aggressive
Life Cycle Fund III	85%	15%	40%	60%	10,515	Conservative
Life Cycle Fund IV	78%	22%	40%	60%	11,011	Conservative
Generic 100/0	100%	0%	100%	0%	7,482	Aggressive
Generic 90/10	90%	10%	90%	10%	8,343	Aggressive
Generic 80/20	80%	20%	80%	20%	9,293	Aggressive
Generic 70/30	70%	30%	70%	30%	10,338	Aggressive
Generic 60/40	60%	40%	60%	40%	11,487	Moderate
Generic 50/50	50%	50%	50%	50%	12,746	Conservative
Generic 40/60	40%	60%	40%	60%	14,123	Conservative
Generic 30/70	30%	70%	30%	70%	15,625	Conservative

Note: For each of the four Life Cycle funds, asset allocation percentages change linearly from year 1 to year 30. For each of the Generic funds, asset allocation percentages remain constant from year 1 to year 30.

3.1. Model 1

Constant real dollar amounts invested annually are computed based on expected real rates of return for the assumed asset allocation. That is, the targeted portfolio amount (i.e., $1,000,000 in today's dollars) is amortized at the expected real rate of return to obtain the required annual payment. Table 1 displays the level contribution amounts for four life-cycle-like asset allocations and eight generic asset allocations. Note the annual contribution amounts range from $7,482 per year for the strategy with 100% in equities to $15,625 per year for the 30% equities/70% bonds strategy.

Each asset allocation analyzed in Model 1 is amortized at a different expected rate of return, making it difficult to compare risk profiles of the several asset allocation schemes. Therefore, we have modified Model 1 to consider identical contribution amounts for each of the asset allocations considered. We separately consider constant real dollar contributions of $11,000 per year and $16,000 per year.

3.2. Model 2

For each year, we calculate a target end-of-year balances based on amortizing the retirement portfolio target using the expected rate of return for the relevant asset allocation strategy. Next, the contribution amount at the end of year t is established so that the end-of-year t balance equals or exceeds the targeted balance. Note that if the simulated balance exceeds the targeted balance, a zero portfolio contribution is made for that year. Some years may require no payments, whereas other years may require large payments when simulated investment performance has declined significantly. Because Model 2 guarantees meeting or exceeding the investment goal, average real dollar payments over the 30-year period will usually exceed the level real dollar payment computed in Model 1.

4. Data and methods

In all simulation models reported, we use annual returns data from Ibbotson and Associates (2006). Time series data for the years 1926 through 2005 include the following three rates with corresponding means and standard deviations.

Data Series	Mean	Standard Deviation
· Large Cap Stocks	12.30%	20.20%
· Long-Term Corporate Bonds	6.24%	8.51%
· Inflation	3.13%	4.29%

Rates of return for asset categories and rates of inflation are simulated using resampling methods that randomly sample from the 80 years of data. To begin the sampling process for a given trial, a year is randomly sampled from the range 1926 through 2005. For the second and subsequent years in the trial, the current year is incremented by one until 30 years are reached or the year 2005 occurs. If the year 2005 occurs before the 30th year is reached, the next year is obtained by randomly sampling from the years 1926 through 2005, and the process described above is continued. Rates of return for Large Cap Stocks and Long Term Corporate Bonds along with the inflation rate for the sampled year are then applied to end of year balances. One simulation trial includes 30 sample years to model savings/investment for the specified asset allocation strategy. The rates sampled in each trial are applied to both Model 1 and Model 2. The resampling method captures correlation across asset categories and inflation. Observations used in the simulations are sampled directly from the existing set of historical rates and, therefore, do not require assumptions of specific probability distributions such as the normal or lognormal distributions. In all simulation scenarios, portfolios are rebalanced annually.

Our methodology is similar to the method of overlapping periods (Cooley et al., 2003). The advantage of the overlapping periods method is that it implicitly addresses issues of serial correlation and mean reversion. Using the method of overlapping periods applied by Cooley et al. (2003) with the Ibbotson data would limit the number of periods to 22, i.e., those 30 year periods beginning with years from 1955 to 1976. Using other 30-year periods would produce a biased sample. In contrast, our method possesses limited bias with the years 1926 through 1954 occurring with linearly increasing frequency. The years 1955 through 2005, however, occur with equal probability. The early years of the various time series beginning with 1926 display considerable volatility with wide year-to-year fluctuations in rates in contrast to years since 1946, which display lower levels of volatility. Consequently, our methodology does not rule out sequences of years similar to what occurred in the late 1920s and early 1930s, but such occurrences have diminished chances of occurring in our simulation trials. Past behavior of time series as a predictor of future behavior is implicit in nearly all Monte Carlo approaches. The method of resampling used in this paper implicitly addresses several issues such as serial correlation, mean reversion, moving average, and auto-regressive properties common to time series data.

In Model 1, we compute the constant real dollar amount contributed each year by dividing the targeted terminal amount, for example, $1,000,000, by the appropriate future value annuity factor. The future value annuity factor is comprised of 30 future value interest factors defined as follows

k_{ta}, = weighted expected portfolio return for year t based on the asset allocation specified for year t
FV_t = future value annuity factor for year t
$FV_t = (1 + k_{ta})^*FV_{t-1}$ for t = 2,3, . . ., 30 where $FV_1 = 1$

The future value annuity factor used to compute the level contribution amount is the sum of the 30 future value interest factors, that is, $FVAF_{30} = FV_1 + FV_2 + \ldots + FV_{30}$.

Model 2 assumes annual contributions such that the portfolio balance each year equals or exceeds the target portfolio balance for the year. The year-to-year target portfolio balances are based on the real dollar amounts generated by the level contribution amount used in Model 1, assuming that weighted mean rates of return prevail each year. The annual contribution for Model 2 is equal to the amount necessary to achieve the target annual balance.

Table 2 illustrates an example of a mixed asset allocation strategy with corresponding level contribution amounts, future value factors, and target portfolio balances. Note that asset allocations have been specified to produce a real annual contribution of $10,000. Table 3 illustrates a single simulated trial of 30 years for Model 1 based on the asset allocation in Table 2. The simulated trial of Model 1 is generated as follows. A starting year, for example, 1988, is randomly selected from the range 1926 through 2005. The rates of return for Large Cap Stocks (16.81%) and Long Term Corporate Bonds (10.70%) along with the inflation rate (4.42%) are returned for the selected year.

1. The real dollar contribution of $10,000 becomes the Real End-of-Year Balance for the first year 1988. The corresponding Nominal Annual Contribution and Nominal End-of-Year Balance are computed by applying the inflation rate of 4.42% ($10,442).
2. The specified asset allocation (84.19%/15.81%) from Table 2 is applied to the Nominal End-of-Year Balance to obtain the Large Cap and LT Corp Bonds balances of $8,791 and $1,651, respectively.
3. The year is incremented by 1 to 1989 and the corresponding rates of return for Large Caps and Long Term Corporate Bonds are returned along with the inflation rate for 1989.
4. The nominal annual contribution is computed by applying the cumulative two-year inflation factor of 1.0928 to obtain the nominal contribution of $10,928.
5. The previous year (1988) balances for Large Caps and LT Corp Bonds are updated to reflect the current year's rates of return. Large Caps earned 31.49% in 1989, which yields a balance of $11,559 (1.3149 X 8,791). Long Term Corporate Bonds earned 16.23% in 1989, which yields a balance of $1,919 (1.1623 X 1,651). The total nominal balance at the end of 1989 is $24,406 (new contribution of $10,928 plus $11,559 plus $1,919). The portfolio is then rebalanced by applying the asset allocation of 82.98% Large Caps ($20,252) and 17.02% Long Term Corporate Bonds ($4,154).

Table 2 Example of asset allocation with weighted real returns

Year	Large caps	LT corp bonds	Weighted real ROR	FV_t	Target balance
1	84.19%	15.81%	7.97%	1.0000	10,000
2	82.98%	17.02%	7.89%	1.0797	20,797
3	81.79%	18.21%	7.82%	1.1649	32,445
4	80.63%	19.37%	7.76%	1.2560	45,006
5	79.49%	20.51%	7.69%	1.3534	58,540
6	78.38%	21.62%	7.62%	1.4575	73,115
7	77.30%	22.70%	7.56%	1.5686	88,802
8	76.23%	23.77%	7.50%	1.6872	105,674
9	75.20%	24.80%	7.44%	1.8137	123,812
10	74.19%	25.81%	7.38%	1.9486	143,298
11	73.20%	26.80%	7.32%	2.0924	164,222
12	72.24%	27.76%	7.26%	2.2455	186,677
13	71.30%	28.70%	7.21%	2.4086	210,764
14	70.39%	29.61%	7.15%	2.5823	236,586
15	69.50%	30.50%	7.10%	2.7670	264,256
16	68.64%	31.36%	7.05%	2.9635	293,892
17	67.80%	32.20%	7.00%	3.1725	325,617
18	66.99%	33.01%	6.95%	3.3947	359,563
19	66.20%	33.80%	6.91%	3.6307	395,871
20	65.44%	34.56%	6.86%	3.8816	434,687
21	64.70%	35.30%	6.82%	4.1480	476,166
22	63.99%	36.01%	6.78%	4.4309	520,476
23	63.30%	36.70%	6.74%	4.7313	567,788
24	62.64%	37.36%	6.70%	5.0501	618,289
25	62.00%	38.00%	6.66%	5.3884	672,172
26	61.39%	38.61%	6.63%	5.7473	729,646
27	60.80%	39.20%	6.59%	6.1281	790,927
28	60.24%	39.76%	6.56%	6.5321	856,248
29	59.70%	40.30%	6.53%	6.9605	925,853
30	59.19%	40.81%	6.50%	7.4147	1,000,000

Note: Rates: Large Caps 12.30%; Corp Bonds 6.24%; Inflation 3.13%.
Asset allocation percentages change linearly from year 1 to year 30. Constant real dollar payment is $10,000 per year based on $1 million target and expected rates of return.

6. Steps 3 through 5 are then repeated until 30 years have been included or the year 2005 is reached. If 2005 is reached, the subsequent year is obtained by randomly selecting a year from the 1926 through 2005 range. Note that 2005 is reached in year 18 and random sampling returns 1952 for year 19.

As stated previously, Table 1 displays 12 different asset allocations that we analyze. Four of these schemes (Fund I, Fund II, Fund III, and Fund IV) are patterned after life-cycle allocations offered by various financial service companies. For each of the four life-cycle-type allocations, the shift from equities to bonds is linear over the 30 years. The remaining eight, generic asset allocation schemes assume fixed percentages of equities and corporate bonds throughout the entire 30 years. The life-cycle allocations specify percentages in equities and bonds each year whereas the generic asset allocation portfolios are rebalanced each year.

Table 3 Simulated trial of Model 1

Year	Simulated year	Large caps	LT corp bonds	Inflation	Cumulative inflation factor	Real end-of-year balance	Nominal end-of-year balance	Large cap (nominal)	Corp bonds (nominal)	Annual payment (nominal)
1	1988	16.81%	10.70%	4.42%	1.0442	10,000	10,442	8,791	1,651	10,442
2	1989	31.49%	16.23%	4.65%	1.0928	22,334	24,406	20,252	4,154	10,928
3	1990	−3.17%	6.78%	6.11%	1.1595	30,737	35,641	29,151	6,489	11,595
4	1991	30.55%	19.89%	3.06%	1.1950	48,357	57,787	46,594	11,193	11,950
5	1992	7.67%	9.39%	2.90%	1.2297	60,756	74,709	59,389	15,319	12,297
6	1993	9.99%	13.19%	2.75%	1.2635	75,425	95,297	74,696	20,601	12,635
7	1994	1.31%	−5.76%	2.67%	1.2972	83,303	108,061	83,527	24,534	12,972
8	1995	37.43%	27.20%	2.54%	1.3302	119,760	159,300	121,441	37,859	13,302
9	1996	23.07%	1.40%	3.32%	1.3743	146,683	201,590	151,591	49,999	13,743
10	1997	33.36%	12.95%	1.70%	1.3977	195,046	272,612	202,240	70,372	13,977
11	1998	28.58%	10.76%	1.61%	1.4202	247,986	352,186	257,797	94,389	14,202
12	1999	21.04%	−7.45%	2.68%	1.4582	283,887	413,977	299,045	114,932	14,582
13	2000	−9.11%	12.87%	3.39%	1.5077	276,320	416,603	297,039	119,564	15,077
14	2001	−11.88%	10.65%	1.55%	1.5311	267,371	409,359	288,140	121,218	15,311
15	2002	−22.10%	16.33%	2.38%	1.5675	243,159	381,149	264,904	116,246	15,675
16	2003	28.70%	5.27%	1.88%	1.5970	300,116	479,272	328,969	150,304	15,970
17	2004	10.87%	8.72%	3.26%	1.6490	330,274	544,628	369,269	175,359	16,490
18	2005	4.91%	5.87%	3.42%	1.7054	346,019	590,107	395,312	194,795	17,054
19	1952	18.37%	3.52%	0.88%	1.7204	399,196	686,787	454,671	232,115	17,204
20	1953	−0.99%	3.41%	0.62%	1.7311	408,709	707,512	462,999	244,513	17,311
21	1954	52.62%	5.39%	−0.50%	1.7224	569,859	981,545	635,090	346,455	17,224
22	1955	31.56%	0.48%	0.37%	1.7288	694,658	1,200,931	768,485	432,445	17,288
23	1956	6.56%	−6.81%	2.86%	1.7783	697,132	1,239,676	784,757	454,919	17,783
24	1957	−10.78%	8.71%	3.02%	1.8320	662,146	1,213,022	759,849	453,173	18,320
25	1958	43.36%	−2.22%	1.76%	1.8642	832,033	1,551,074	961,721	589,354	18,642
26	1959	11.96%	−0.97%	1.50%	1.8922	887,505	1,679,301	1,030,940	648,362	18,922
27	1960	0.47%	9.07%	1.48%	1.9202	917,710	1,762,155	1,071,450	690,704	19,202
28	1961	26.89%	4.82%	0.67%	1.9330	1,087,873	2,102,890	1,266,798	836,092	19,330
29	1962	−8.73%	7.95%	1.22%	1.9566	1,062,210	2,078,334	1,240,831	837,503	19,566
30	1963	22.80%	2.19%	1.65%	1.9889	1,206,435	2,399,473	1,420,259	979,215	19,889

Note: The single trial includes simulated rates for 30 years. The asset allocation data are displayed in Table 6. The constant real dollar contribution is $10,000 per year.

4.1. Model 1 results

The inherent risk in any plan that considers level annual payments that are based on expected rates of return is revealed in Fig. 1.

- The probability of failing to reach the target of $1,000,000 exceeds 50% for all 12-asset allocations considered. For the four life-cycle type allocations and the four most aggressive generic strategies, the probabilities of failing to reach the target range from 51% to 73%.
- The generic strategy of 70% equities and 30% bonds (heretofore referred to as 70/30) is superior to all four of the life-cycle style allocations in reaching the target amount (49% probability), and in reaching 75% of the target goal (67% probability). The 70/30

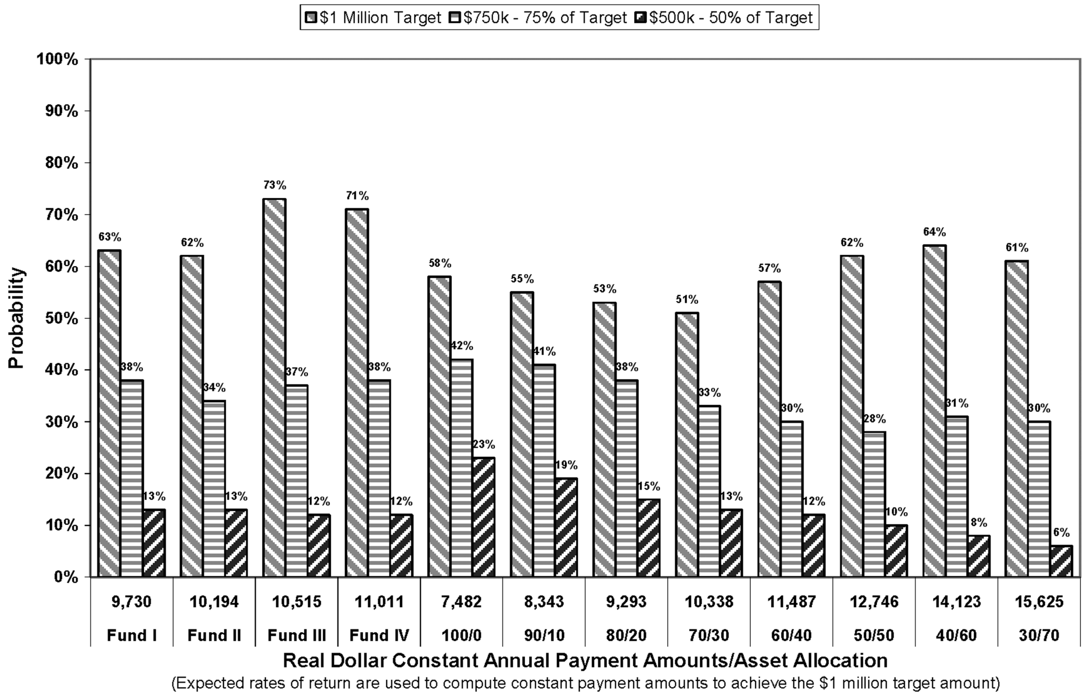

Fig. 1. Probability of failing to reach targeted amounts for Model 1.

portfolio has a greater likelihood of reaching targeted goals than both the aggressive and conservative life-cycle funds.

Figure 2 and Table 4 provide Model 1 comparisons of the 12 asset allocation schemes assuming a contribution amount of $11,000 per year for each of the schemes.

- The 100% equity allocation realizes a 39% probability of failure to meet the target, a 24% probability of failing to meet 75% of target, and a 4% probability of failing to meet 50% of target. Each of these failure probabilities is less than the corresponding probabilities for the four life-cycle funds.
- The four life-cycle schemes, the 80/20 generic allocation, and 70/30 generic allocation all achieve similar probabilities of failure ranging from 10% to 12% to meet 50% of target. Note that the two most aggressive generic allocations (100/0 and 90/10) fail to meet 50% of the target with probabilities of 4% and 5%, respectively. The least aggressive generic allocations of 60/40, 50/50, 40/60, and 30/70 demonstrate substantial risk with failure probabilities for 50% of target ranging from 14% to 23%.
- The 70/30 generic asset allocation achieves failure probabilities approximately equal to or less than the four life-cycle allocations.
- Table 4 reveals that the three most aggressive generic allocations (100/0, 90/10, and 80/20) achieve median balances exceeding the $1,000,000 target. Moreover, first quartile balances for these three schemes exceed the first quartile balances of all of the other nine allocation schemes considered.

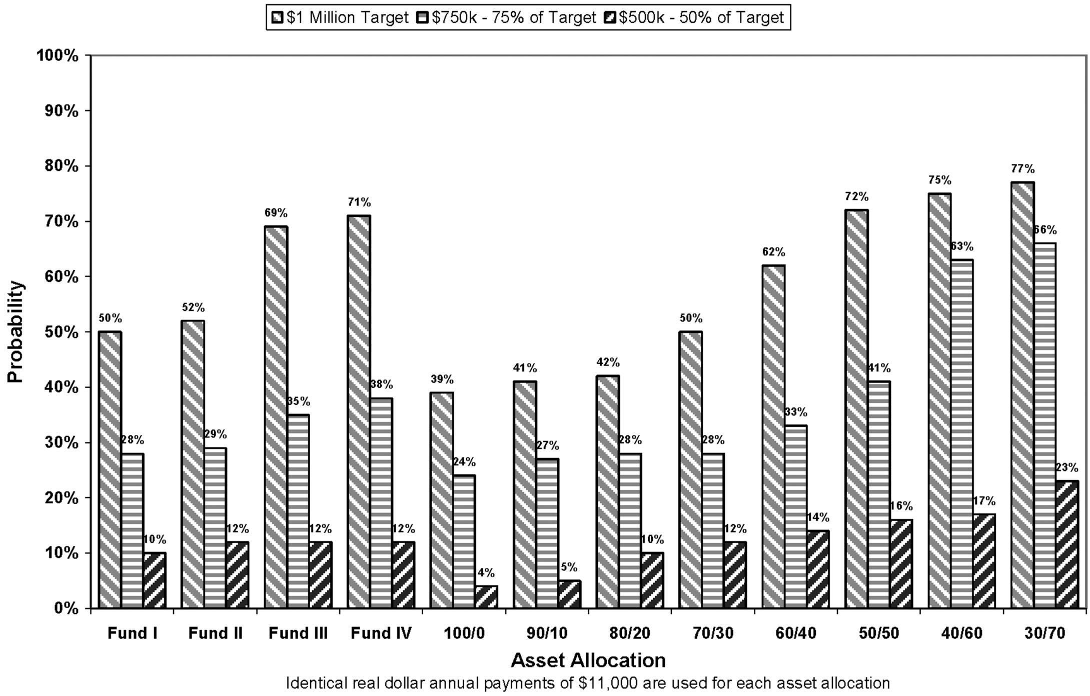

Fig. 2. Probability of failing to reach targeted amounts based on $11,000 per year real dollar contribution.

- Even though the three most aggressive generic asset allocations demonstrate high volatility with 30 year balance standard deviations exceeding $400,000, these three asset allocations demonstrate superior risk performance from a VAR perspective.

Increasing the level contribution amount from $11,000 per year to $16,000 per year reduces failure-to-reach target probabilities to a range of 20% to 29% for the nine most aggressive

Table 4 Summary statistics for 30 year balances with $11,000 per year

Asset allocation	Mean	Standard deviation	5th Percentile	1st Quartile	Median	3rd Quartile	95th Percentile
Fund I	989,164	351,906	450,522	679,973	993,855	1,245,916	1,567,878
Fund II	965,206	344,434	437,808	668,921	967,023	1,203,937	1,543,703
Fund III	885,574	312,397	423,567	635,687	854,753	1,111,752	1,377,822
Fund IV	864,222	307,901	414,376	627,495	815,395	1,092,915	1,359,022
100/0	1,303,259	574,936	503,901	762,868	1,318,805	1,749,631	2,197,197
90/10	1,189,538	483,740	481,279	730,414	1,244,462	1,593,760	1,898,064
80/20	1,086,118	414,380	470,278	702,422	1,126,413	1,400,589	1,717,383
70/30	992,126	363,929	438,464	657,638	996,997	1,256,792	1,612,725
60/40	906,731	328,944	407,165	633,793	924,026	1,128,872	1,480,935
50/50	829,150	305,626	377,042	606,994	790,133	1,062,671	1,377,164
40/60	758,653	290,269	349,307	553,533	683,020	989,474	1,255,830
30/70	694,563	279,719	323,549	501,872	592,934	921,045	1,154,545

Note: Constant real dollar contributions of $11,000 per year are assumed for each asset allocation. Statistics are based on 1000 trials.

Table 5 Summary statistics for Model 2, 30 year balances

Asset allocation	Mean	Standard deviation	5th Percentile	1st Quartile	Median	3rd Quartile	95th Percentile
Fund I	1,182,437	173,487	1,000,000	1,006,671	1,154,748	1,333,791	1,472,083
Fund II	1,184,318	178,293	1,000,000	1,008,149	1,150,394	1,344,198	1,486,027
Fund III	1,145,946	168,867	1,000,000	1,000,000	1,078,252	1,234,210	1,491,585
Fund IV	1,149,641	175,494	1,000,000	1,000,000	1,072,879	1,233,357	1,506,308
100/0	1,307,423	317,924	1,000,000	1,043,410	1,216,942	1,541,118	1,958,232
90/10	1,269,905	262,693	1,000,000	1,050,025	1,195,632	1,432,674	1,784,349
80/20	1,237,105	222,754	1,000,000	1,025,955	1,184,014	1,399,089	1,738,337
70/30	1,211,220	199,266	1,000,000	1,001,579	1,184,978	1,378,891	1,585,701
60/40	1,191,626	194,112	1,000,000	1,011,805	1,133,770	1,369,983	1,525,520
50/50	1,179,660	202,495	1,000,000	1,000,168	1,096,406	1,299,881	1,557,973
40/60	1,176,393	215,551	1,000,000	1,000,000	1,066,683	1,338,095	1,588,839
30/70	1,179,047	227,527	1,000,000	1,000,000	1,039,970	1,308,938	1,627,650

Note: Annual real dollar contributions are set to meet end of year targets. Statistics are based on 1,000 trials.

asset allocations, whereas the three least aggressive generic allocations (50/50, 40/60, and 30/70) failure probabilities remain high at 34%, 50%, and 60%, respectively.

For further insight, the performances of each of the life-cycle allocation schemes are compared with corresponding allocations that reverse the allocation patterns. For example, the original allocations for Fund IV, which begin with 78/22 and end with 40/60, are reversed to begin with an allocation of 40/60 and end with an allocation of 78/22 in year 30. For the conservative Funds III and IV, greater ending balances are achieved by the reverse allocations at the first quartile, median, third quartile, and 95 percentile levels. Even at the fifth percentile, the reverse funds achieved balances that are only 2.1% to 2.8% less compared with the balances for the original allocations. In other words, asset allocations that shift to equities and away from bonds as the planning period approaches, provide performance that may be equal to or even superior to the original allocation schemes.

4.2. Model 2 results

Model 2 offers a contribution strategy that is guaranteed to achieve the target by adjusting annual contributions. Each year the contribution amount is adjusted to meet a target that represents the account balance, assuming that mean real rates of return are realized each year. No contributions are made in years when the account balance exceeds the target balance. Table 5 displays 30 year balance statistics for Model 2. Note that the three most aggressive generic allocations (100/0, 90/10, 80/20) demonstrate larger balances at the first quartile, median, third quartile, and 95th percentile levels compared to the four life-cycle allocations. The higher standard deviation of these four generic allocations reflects the upside potential for large ending balances.

Equivalent annual payment (EAP) amounts are computed for each simulation trial for Model 2 by converting simulated ending balance amounts to equivalent annual annuity amounts. Expected real rates of return are used to compute both present values of contributions and corresponding EAPs. Table 6 displays summary statistics for the equivalent annual payment amount for each of the 12 portfolios. Note the following points.

Table 6 Summary statistics for equivalent annual payment

Asset allocation	Mean	Standard deviation	5th Percentile	1st Quartile	Median	3rd Quartile	95th Percentile	Model 1 level payment
Fund I	12,533	3,755	7,709	9,154	11,745	15,516	19,508	9,730
Fund II	12,923	3,879	7,836	9,571	12,044	15,902	20,132	10,194
Fund III	13,716	4,192	7,923	10,625	12,922	16,768	21,631	10,515
Fund IV	14,158	4,336	8,034	10,969	13,357	17,305	22,306	11,011
100/0	10,144	2,813	6,269	7,817	9,556	12,586	14,750	7,482
90/10	10,953	3,097	7,095	8,144	10,195	13,692	16,275	8,343
80/20	11,843	3,431	7,525	8,817	11,100	14,632	18,015	9,293
70/30	12,852	3,796	8,014	9,484	11,904	16,062	19,744	10,338
60/40	13,996	4,236	8,105	10,893	12,982	17,707	21,423	11,487
50/50	15,311	4,731	8,297	11,857	14,674	19,668	23,464	12,746
40/60	16,856	5,287	8,623	12,505	16,511	21,449	25,666	14,123
30/70	18,647	5,957	8,977	13,420	18,941	23,124	28,022	15,625

Note: The equivalent annual payment (EAP) is computed by dividing ending balance amounts by corresponding annuity factors. The above table displays simulation summary statistics bases on 1,000 trials. Model 1 level payments are displayed for comparison.

- The mean EAP amounts exceed the level annual payment contributions for Model 1 by amounts ranging from $2509 ($13,996 compared with $11,487) for the generic 60/40 asset allocation to $3201 greater for the Fund III asset allocation. Nevertheless, the generic 100/0 equities asset allocation realizes the lowest mean equivalent annual payment of $10,144. The next lowest mean equivalent annual payment amount is $10,953 for the 90/10 asset allocation. The more aggressive the portfolio, the less is the mean equivalent annual payment.
- The three most aggressive generic asset allocations realize the least EAP standard deviations. In addition, EAP amounts for these most aggressive, generic asset allocation portfolios are less at the fifth percentile, first quartile, median, third quartile, and 95th percentile levels compared with the other nine asset allocations.
- The three most conservative generic allocation strategies are characterized by both large mean EAPs and large EAP standard deviations.

Simply put, although an investor using the Model 2 strategy will achieve the one million dollar target, she will frequently have to invest significant amounts of dollars annually to do so. The most aggressive, equity-heavy portfolio is more likely to achieve a higher end result than the more conservative, bond-heavy portfolios.

5. Discussion

The so-called conventional wisdom of life-cycle allocations calls for portfolios with high levels of equities in the beginning years to be followed by decreasing the equity levels as the investor nears retirement. Our simulated portfolios reveal that simple aggressive generic portfolios, that is, portfolios that maintain constantly high proportions of equities throughout the entire 30-year investment cycle, outperform all of the simulated life-cycle allocations. Indeed, a portfolio with 70% equities and 30% bonds performs about the same as the most aggressive life-cycle portfolio considered, and the 70/30 allocation outperforms the other three life-cycle portfolios considered. Additionally, it is possible that the performance of such life-cycle funds is further diminished by the transaction costs and management fees associated with some life-cycle funds; a cost we do not consider in our simulations. We are neither advocating nor criticizing the life-cycle-like funds, but instead observe that with both Model 1 and Model 2, the generic 70/30 portfolio behaves similarly to the target funds. Recently, life-cycle funds appear to be moving to allocations that are more aggressive. For example, Laise (2006) reports that fund managers are structuring life-cycle portfolios with allocations that have increasing exposure to international equities. This movement toward more aggressive allocations is consistent with our findings. Funds that are more equity-heavy are more volatile, but over a 30-year period, are more likely to achieve one's specific retirement target than less aggressive funds.

A number of studies, such as Booth (2004) and Kaplan (2000), have noted that portfolio values tend to follow the lognormal distribution. Specifically, if rate of return is normally distributed, then one plus the rate of return is lognormal. In the conventional approach to risk, riskier portfolios are characterized by greater volatility. The right-skewness of the lognormal distribution suggests that much of any increased volatility may be captured in the right hand tail of the distribution.

Model 2 offers an approach to reducing the risk associated with reaching a target portfolio amount. In the early years, only modest adjustments are required to keep the portfolio at or above its targeted amounts. Adjustments in later years may be substantial. For example, consider a portfolio with an asset allocation of 80% or 90% in equities that is hit with a negative 20% rate of return in the 28th or 29th year. Contributions could exceed $100,000 in real dollars to increase the portfolio to its targeted value.

In both Model 1 and Model 2, measurements of risk are based on the VAR concept. Establishing probabilities of failure to achieve specific portfolio values is more important to the investor than measures of volatility.

6. Conclusion

To state that risk is an endemic component of retirement planning ventures on belaboring the obvious; stock and bond prices fluctuate, as do inflation rates. As a result, there is no certain way to reach portfolio target values. Strategies abound for retirement planning, and a bevy of life-cycle mutual funds appear to be gaining appeal because of the automatic portfolio reallocation from more volatile, aggressive assets (stocks) to less aggressive assets (bonds and cash) over time.

The simulations and models presented in this paper reveal three findings that deserve careful attention when planning retirement portfolios. First, there is little evidence that life-cycle funds either enhance or detract from overall performance over a 30-year investment period. Rather, simulated portfolios of four different life-cycle accounts perform similarly to generic 70/30 asset allocated portfolios. Given this similarity of performance, the merits of life-cycle accounts may lie in their passive, hands-off approach. Unlike the similarly performing generic 70/30 or 80/20 portfolios we simulate, the life-cycle models do not require investors to reassess and reallocate annual profits, dividends, and capital gains. That written, we emphasize that the least aggressive of the life-cycle portfolios consistently underperform relative to simpler generic portfolios. This finding suggests that those investors pursuing life-cycle-like portfolios choose the more aggressive options, as the upside potential is far greater than the downside risk. This finding is also consistent with recent newspaper articles that advocate a steady investment in equities, even as one approaches retirement age [see Opdyke (2006); Mincer (2006)]. Furthermore, retirees in good health with significant wealth amassed who seek capital appreciation, not merely capital preservation, throughout their retirement years, may find non-aggressive life-cycle funds with low equity allocations unable to provide satisfactory growth.

Second, our simulations and analysis seek to spark a discussion about what constitutes risk, and the need to evaluate one's risk tolerance throughout the retirement planning years. Does one prefer to make regular investments annually (perhaps through a payroll deduction on a monthly basis), or is the investor willing to review their investments annually, and decide to increase significantly their retirement investments to achieve a long-term 'goal' that may be a decade or two away? We contrast two strategies for making annual contributions to retirement portfolios. Both strategies have inherent risks. Other strategies similar to Model 2 may provide relatively high probabilities of achieving target values without requiring extremely large contributions in later years. Note how risk has not been mitigated, nor has it been accelerated. Rather, the risk remains in (1) not reaching one's goal, (2) achieving the goal by having to invest extraordinary sums annually, or (3) coming close to reaching one's retirement goal, only to have a sudden market downturn near retirement result in a significant loss of capital just as one approaches retirement age.

Third and finally, we hope that the above analysis stimulates discussion in both the scholarly and investment communities, as the data suggest that the presumed advantages of minimizing equity allocations over time is a dubious one. All of the life-cycle-like funds we evaluate reduce equity exposure over time. Our findings from the analysis of life-cycle reversed allocations indicate that portfolios that increase equity allocations over time are more likely to yield higher performance with similar downside risk to decreasing equity allocation schemes. We hope that our findings underscore the need to locate the proper pre-retirement investment vehicles. Choosing to adopt a dynamic strategy may require significant contributions. Choosing to adopt a static strategy is likely to yield an outcome that misses a target goal, and the chances of achieving that goal will require a heavy investment in equities.

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