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Optimizing social security benefit initiation and postponement decisions: a sequential approach

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Abstract

The paper supposes, consistent with law, that single or married Social Security beneficiaries view an initiation or postponement decision in terms of a sequential decision process rather than in terms of a single evaluation made at or before the normal retirement age. The question asked at each age, according to the sequential approach introduced in this paper, is not whether to initiate now or postpone until some time in the distant future, but whether to initiate now or postpone for just one year. This paper shows that the opportunity rate of return (or minimum investment yield) required to justify initiation at any eligible age (62 through 69), varies from one eligible retirement age to the next within any cohort group, and at any eligible age across cohort groups. Moreover, while members of a particular cohort group might find it advantageous to initiate benefits at a particular age, say at age 62, early retirement might not be advantageous a year later. This oscillation, uncovered by sequential analysis, could not be so easily demonstrated using an aggregative approach. © 2008 Academy of Financial Services. All rights reserved.

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1. Introduction

The Normal Retirement Age (NRA) for Social Security beneficiaries born between 1943 and 1954 is 66 and their Early Eligibility Age (EEA) is 62. Beneficiaries who elect to initiate

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(start receiving) benefits, at the NRA receive a full retirement benefit (FRB) amount initially, with subsequent monthly payments increased by an annual inflation adjustment factor. The FRB is determined, primarily by the number and amount of payments made into the system before initiation, and is zero for beneficiaries who have made fewer than 40 quarterly payments into the system

A participant may initiate benefits at the beginning of any month starting at age 62 but no later than age 70. The Social Security Administration (SSA) adjusts benefit payments for beneficiaries who initiate before or after reaching NRA, relative to the FRB, using an actuarial adjustment factor that depends on the age at the time of initiation. The maximum benefit reduction (penalty) is 25% when benefits are initiated at age 62 (the EEA) and the maximum increase (premium) is 32% if initiation is postponed until age 70.

The optimal age at which to begin receiving Social Security benefits depends on a myriad of factors. For example, an individual who arrives at the EEA with little or no savings outside the Social Security system and diminished earnings capacity may be more concerned about satisfying current needs than maximizing future value. An exceedingly high interest rate assumption might be required, moreover, to justify the postponement of benefits by an unmarried participant who is in ill health, but given the same circumstances there may be valid justification for a married participant to do so. Approximately 60% of eligible beneficiaries initiate benefits before reaching the NRA, but the proportions vary by gender and marital status [see Table 2 in Munnell and Soto (2007)].

A beneficiary who reaches the EEA, in good health and with adequate income, by contrast, should view an initiation decision as an investment decision that involves a tradeoff between lower lifetime annuity payments for a longer period or higher benefit payments for a shorter period–this tradeoff may carry important financial and future wealth implications. The initiation versus postponement of benefits decision, its tradeoffs, and financial implications are the subjects of a number of recently published papers. It is generally agreed that an initiation versus postponement of benefits decision may have significant consequences, but there is less agreement about how to model the problem or measure its financial implications.

For example, failure to account for the time value of money, (i.e., treating money as a "free" good), causes early opportunity costs incurred by postponement of benefits to be weighted the same as distant cash benefit inflows, thus resulting in a systematic bias in favor of early initiation. By contrast, discounting at *any* positive rate causes the benefits forgone by postponement to be weighted more heavily than later cash inflows, and thus more time is required to recover any foregone benefits.

By law, benefits are paid only to living beneficiaries. Thus, the anticipated future benefits should be weighted by the survival probabilities–the probability that the beneficiary is alive and the benefits will actually be received. Because the survival probabilities decrease monotonically with age, the weights applied to early and later cash flows differ even more, causing the breakeven age to advance further still. The results of any present value or net present value calculation, however, is determined not only by its discount rate assumption and cash flow definitions, but also by the time horizon over which payments are assumed to be made.

One approach for rationalizing the problem of uncertain future interest rates of return versus a fixed interest rate assumption is to discount by many alternative rates. However, as luck or design would have it, for any one or series of cash outflows that is followed by a stream of equal or unequal cash inflows, a single discount rate can always be found that causes the net present value to equal zero;[1] this rate is known as the internal rate of return (IRR). A second approach for rationalizing the problem of uncertain future interest rates of return is to compute this IRR, and then contrast it with plausible market rate of return scenarios that may apply at the time of the decision

An internal rate of return calculation, however, is nothing more than the iterative solution of a net present value problem, and thus is determined in part by its time horizon assumption, often represented by a median life expectancy statistic– or 50th percentile of a cumulative life expectancy distribution. One approach to resolving inconsistencies resulting from the use of a single median life expectancy is to repeat the calculations of present value, net present value, internal rates of return, or breakeven point, as the case may be, using median life expectancy of Social Security beneficiaries who initiate benefits at certain critical ages, such as 62, 66, or 70. This approach falls short of the mark, however, because Social Security beneficiaries may elect to initiate benefits at the beginning of any month starting at age 62 until age 70 is reached, not simply at ages 62, 66, or 70.

By definition of a median, 50% of the population will not survive to their median life expectancies and 50% will live beyond. The use of a median life expectancy as the limit of summation in a rate setting calculation by an annuity provider, such as the SSA, may be valid as the law of large numbers applies in the aggregate. From the vantage point of an individual single or married participant seeking to maximize the expected value of a stream of future benefits, however, optimal solution of an initiation of benefits decision calls for a quite different stochastic modeling approach.

This paper breaks new ground by modeling single or married participant's optimizing initiation/postponement tradeoffs as a sequential decision problem. Every point between ages 62 and 70 at which a participant may initiate benefits is viewed as a decision point provided that one has not already initiated. The question asked at each decision point is not whether to initiate now or postpone until the next critical age, say 66 or 70, but whether to initiate now or postpone for one more period. In the context of the model, as in the real word, if the decision at any point is to postpone, then the beneficiary is faced with the same alternatives in the next period, until reaching age 70. Thus, implicitly or explicitly, a beneficiary revisits this sequential problem at every eligible age (or decision point) starting at age 62, until the decision is made to initiate benefits or age 70 is reached–whichever comes first.

In our model, the opportunity cost (or investment equivalent) corresponding to a decision to delay the start of benefits payments at any decision point results from the passing of just one benefit payment, not a stream of benefits, and can thus be evaluated at the margin. The focus on marginal cost/benefit tradeoffs in this paper may serve to overcome aggregation affects that we believe are implicit in results reported. Longevity is represented in the model by weighting each future benefit that would follow initiation at a particular decision point by the probability that the benefit payment will be received. The time horizon problem is resolved by setting the limit of summation in all iterative internal rate of return calculations equal to 100, which at the time of this writing is the maximum age for which survival probability data are available.[2]

The results show that the relative magnitudes of the actuarial internal rates of return across single male, single female, and married couple cohort groups vary with age, and move up or down in lockstep with each other and with annual changes in a delayed retirement credits that are calculated on the basis of the SSA's schedule of age benefit adjustment factors. It is also observed that a participant who might be advised to initiate benefits at one eligible initiation of benefits age might not be in the next period. This result, which is fully explained in the paper, could not be uncovered using an aggregative model.

2. Literature review

There is a broad and growing literature on the subject of optimization of Social Security benefit initiation and postponement decision, but in this section we will cite only those papers whose findings we exploit in one way or another, or use for purpose of contrast. Detweiler (1999) considers multiple real rates return and investment scenarios. He defines negative cash flow as being created by initiation at age 62 and positive cash flow as being created by initiation at the NRA, which at the time of his study was age 65, and breakeven death age as the time of death at which a net present value to life expectancy function would go to zero. Based on this logic he calculates breakeven death ages under a variety of different real interest rate assumptions, and the probability of dying at or before one's breakeven death age using cumulative distribution mortality data taken from Bell et al. (1992). Restricting his analysis to individuals who will invest benefit awards rather than consume them, and who feel comfortable managing their own investments, he concludes that a male participant whose expected real rate of return on investments is in excess of approximately 2.25% should probably initiate benefits at age 62, but that a female would require an expected real rate of return in excess of approximately 4.5% to justify doing so.

Like Detweiler (1999), Spitzer (2006) considers multiple real rates return, but he also attempts to account for the stochastic nature of rates of return, and their relationship to alternative financial market conditions and asset allocation strategies. He performs breakeven analyses for unmarried single male and single female beneficiaries, born between 1943 and 1954, contemplating initiation at age 62 or 66. His results indicate that postponement is more easily justified for female than male beneficiaries because of longer life expectancy, but for either group–all other things equal–the breakeven initiation age is later the higher discount rate and/or the later the participant's age at initiation.

McCormack and Perdue (2006) first consider the case of single male and female beneficiaries, born in 1943, who at age 62 will decide whether to initiate at age 62, 66, or 70. Their analysis is then extended to married couples. Milevsky, Kwok and Robinson (1997) reminds us, in effect, that the median marks the 50th percentile of a distribution, they call their reader's attention to the implications and shortcomings of considering only the 50th percentile of a cumulative life expectancy distribution in a present value to life expectancy model. To overcome these shortcomings they calculate internal rates of return using a present value to life expectancy model, but model life expectancy on the basis of five different percentiles of cumulative life expectancy distributions for single males, single, females, and married couples, rather than just the 50th percentile. They explicitly assume, however, that, in each case, the participant will live to an age commensurate with life expectancy, receive benefits to that date, and nothing more thereafter. Not surprisingly, internal rates of return, based on a present value to life model, vary with life expectancy. The internal rates of return they report are zero where life expectancy is based on the 10th and 20th percentiles of a cumulative life expectancy distribution, positive at the median except for a male age 66 contemplating postponement until age 70, and positive when life expectancy is based on 70th and 90th percentiles of a cumulative life expectancy distribution. McCormack and Perdue (2006) extended their analysis to consider married couples, but, citing many of the difficulties subsequently outlined by Munnell and Soto (2007), the treatment in this portion of their study is necessarily both more complex and less exhaustive than the simpler cases of single individuals.

Munnell and Soto (2007) discuss initiation decision issues confronting married beneficiaries. A married woman may, for example, receive benefits based on her own contributions starting at age 62, but can trade her benefit for a maximum of 50% of her spouse's adjusted benefit when he initiates, provided that her adjusted benefit does not exceed 50% of his. A married woman at or beyond age 62, by contrast, who would not qualify for benefits on her own nevertheless qualifies to receive a spousal benefit which may be as much as 50% of her spouse's adjusted benefit. Either way, spousal benefits are subject to actuarial adjustment depending on the spouse's age when and if she initiates. Upon a spouse's death, however, regardless of the spouse's age when he passes, she is entitled to 100% of her spouse's benefits in exchange for her own benefit or spousal benefit. The authors argue that a spouse with a living and healthy spouse who is assumed to be the same age as she, will typically be better off initiating early. A spouse, on the other hand, must take into account not only his own expected benefit stream over his lifetime, but also the impact that his age on initiation decision will have on his spouse's spousal and/or contingent benefit streams. Accordingly, the authors argue, that married men should typically initiate later.

Spitzer (2006) and McCormack and Perdue (2006) contrast the financial implications of initiation at age 62 against postponement until age 66, and initiation at 66 against postponement until age 70. They employ somewhat different empirical methods, calculating breakeven points in one case and present values in the other, but both treat future cash flows to life expectancy as constants and discount to life expectancy. We exploit their results and insights in various ways, but extend them by using a more robust life expectancy proxy than median life expectancy to calculate internal rate of return, and by incorporating an actuarial, rather than deterministic accounting, future cash flow definition. In this paper, moreover, a decision to initiate benefits or postpone is viewed in terms of a sequential decision process that begins at age 62, rather than as a dichotomous choice, made at the EEA, whether to postpone until a breakeven initiation age, at age 62, 66 or to age 70.

3. Data, the issue, and the model

By law a participant may initiate benefits at age 62, or, failing to do that, may revisit the problem one month later, and each month after that until the decision to initiate benefits is made or age 70 is reached–whichever comes first. Potentially, therefore, a participant may revisit the problem each month, starting age 62 until one month before age 70. Ideally, one might argue, each monthly incremental cost-benefit tradeoff should be considered, but this would be neither beneficial nor practicable. As SSA age benefit and cost of living adjustments are annual, for example, intermonth present value calculations would differ only as regards a compounding period definition (i.e., monthly vs. annual) and a slightly modified interval of summation. More importantly, however, at least from the vantage point of our model, the survival probability data needed to actuarially adjust future benefit cash flow is available only on an annual basis. Accordingly, this study assumes that the postponement versus initiation decisions are made on an annual rather than monthly basis and that benefits are received once a year rather than monthly.

3.1. The model

The model supposes that initiation versus postponement decisions are made sequentially by revisiting the problem annually, at each decision age, until the decision to initiate benefits is made or, at age 70, is required by the model: Consistent with previously used nomenclature, let SSB_x denote the benefit that would be received by a participant who initiates benefits at age x, $x = 62, 63, \ldots, 70$ [henceforth, age $x \leq 70$]. A participant who initiates benefits at the NRA, $x = 66$, for example, would receive $SSB_{66} =$ FRB. Thus, we may represent the retirement age benefit adjustment factor applied by the SSA at age x, $x = 62, 63, \ldots, 69$ (henceforth, age $x \leq 69$) in terms of ratio, SSB_x / FRB, and define the *delayed retirement credit per dollar* at age x in terms of the relative change in benefits:

$$\lambda_x = (SSB_{x+1} - SSB_x)/SSB_x \tag{1}$$

Initiation at the earliest the retirement age, $x = 62$, would result in the fixed lifetime annuity payment of $SSB_{62} = 0.75*FRB$, whereas initiation at age 63 would result in a higher benefit, $SSB_{63} = 0.8*FRB = (1+\lambda_{62}) * SSB_{62}$. In general, therefore, postponement until any age x, $62 < x < 70$, is tantamount to establishing a present value of future wealth position that is based on $SSB_{x+1} = SSB_x * (1 + \lambda_x)$.

At any age $x \leq 69$, the present value of a participant's guaranteed Social Security benefit wealth, whether or not he or she initiates in that age, has already been established. The opportunity cost of postponing benefits for one more period, therefore, is measured in terms of a single annual foregone benefits SSB_x. However, what does one receive for this foregone benefit?

What one receives in exchange for investing SSB_x to postpone benefits for one year, at any age $x \leq 69$, is an incremental Social Security benefit wealth position at age $x+1$ that is determined by the present value of a difference in benefits ($SSB_{x+1} - SSB_x$), and not the higher benefit SSB_{x+1} as such.[3] To see this, let PV_x and PV_{x+1} represent \$1.00 annual annuity present values.

$$PV_x = \Sigma_{t=x} (1 + k)^{-t}$$

$$PV_{x+1} = (1 + \lambda_x) * \Sigma_{t=x+1} * (1 + k)^{-t} = \Sigma_{t=x+1}(1 + k)^{-t} + \lambda_x \Sigma_{t=x+1}(1 + k)^{-t}$$

Thus, the net present value per dollar of investing PV_x to get back PV_{x+1} is given by:

$$
\begin{aligned}
NPV &= PV_{x+1} - PV_x \\
&= [\Sigma_{t=x+1}(1 + k)^{-t} - \Sigma_{t=x}(1 + k)^{-t}] + \lambda_x \, \Sigma_{t=x+1}(1 + k)^{-t} \\
&= -1 + \lambda_x \, \Sigma_{t=x+1}(1 + k)^{-t}
\end{aligned}
$$

Setting *npv* to zero we obtain:

$$1 = \lambda_x \, \Sigma_{t=x+1}(1 + \mathrm{IRR})^{-t} \tag{2}$$

However, benefits are paid only to live individuals. Let ${}_xS_{bt}$ represent the probability that a beneficiary who initiates at age $x \leq 70$, survives for at least t years after the date of initiation x, where the subscript b is replaced by m or f below to distinguish between male or female beneficiaries. Inserting ${}_xS_{bt}$ in Eq. (2) to actuarially adjust all future cash flow and obtain an expression for calculating actuarially adjusted *IRR*, we get:

$$1 = \lambda_x \Sigma_{t=x+1} \, {}_x\mathrm{S}_{bt} * (1 + \mathrm{IRR})^{-t} \tag{3}$$

We use this definition throughout the remainder of the paper to calculate actuarially adjusted yields, *IRR,* on postponement decisions made by single males or single females. Henceforth in this paper, *IRR* should be understood to represent actuarial internal rate of return–which, it should also be understood, is always lower than the corresponding accounting internal rate of return.

Multiplication of both left and right-hand sides in Eqs. (2) or (3) by any arbitrary constant would have no affect on *IRR*. Spitzer (2006) uses similar logic to show that, for purpose of contrast, (1) when discount rates are stated in real terms there is no need to also inflation adjust the benefit streams, and (2) that the rate at which benefits are taxed will have no affect on an initiation decision, provided that the additional income does not place a beneficiary in a higher tax rate bracket. We exploit these results below, but otherwise modify the methods used in this, and other, previous papers.

Discounting to any time horizon in Eq. (3), is tantamount to assuming, that the benefit due at any age $x + t$, starting at age $x + 1$ to an age consistent with that time horizon (or limit of summation in the expression), is conditional on S_{bt}, the (survival) probability that the beneficiary will survive at least t years following the initiation of benefits at age x. Because survival probability data, ${}_xS_{bt}$, are available only through age 100, we set the limit of summation to 100 in all *IRR* calculations in this paper.[4] Substitution of the appropriate survival probability function, ${}_xS_{mt}$ for males and ${}_xS_{ft}$ for females, in Eq. (3), yield expressions used in this paper to calculate (by method of iteration) one-period *IRRs* on postponement decisions made at each age $x \leq 69$ by single male and female beneficiaries, respectively. Treatment of married couples presents us with a more formidable modeling problem, however.

While not an exhaustive treatment, various complications, found by others, that arise when considering postponement decisions by married beneficiaries are explained in Section 2. Summarizing briefly, a spouse does not necessarily earn less than her spouse, and is not necessarily younger or older than he. The relevant joint survival probability function for a couple, $f({}_xS_{mt} \cap {}_xS_{ft})$, therefore, must depend somehow on their relative ages. A spouse's qualifications for spousal and/or survival benefits, moreover, are subject to a myriad of rules and exceptions to rules. The number of permutations and combinations of these factors, all other things equal, is too large to be treated exhaustively in a single paper, and "all other things" are not necessarily equal. Therefore, some limiting assumptions are necessary.

On average, women live longer than men and earn less. Thus, although we do not explicitly deal with the case where the spouse's adjusted retirement benefit is less than his spouse's, the *IRRs* that we report would be no different if the gender subscripts were reversed. Other assumptions are more restrictive however. We assume that:

1. The spouse and spouse are the same age, retire at the same time, that neither will be subject to an earned income penalty when benefits are initiated; and that
2. The spouse's adjusted benefit does not exceed 50% of her spouse's, and thus that she is entitled to a spousal benefit equal to 50% of her spouse's when he initiates benefits and although he is alive, and, if she is the second to die, to a contingent benefit equal to 100% of his adjusted benefit thereafter.

Consistent with the first assumption, it is also possible that the spouse's adjusted retirement benefit is greater than 50%, but less than, her spouse's. The implications in this case would be that same as for that of a single female while the spouse is alive, but would not be the same after his death if she is the second to die. The model, as presently formulated, does not account for this possibility, but we take account of it in the discussion.

Consistent with these assumptions, and the cases to be represented in the model, let:

1. ${}_xS_{m\,t} * {}_xS_{ft}$ denote the joint probability that both spouse and spouse survive for at least t years after the initiation of benefits at identical ages $x \leq 70;$
2. ${}_xS_{m\,t} * (1 - {}_xS_{ft})$ denote the joint probability that the spouse survives for at least t years after the initiation of benefits, but that his spouse is not alive at time t; and
3. ${}_xS_{ft} * (1 - {}_xS_{mt})$ denote the joint probability that the spouse survives for at least t years after the initiation of benefits, but that her spouse is not alive at time t.

Substituting variable definitions in Eq. (3) we get:

$$1 = \lambda_x * \Sigma_{t=x+1}\{ [1.5 *{}_xS_{mt} * {}_xS_{ft}] + [{}_xS_{mt} * (1 - {}_xS_{ft}) + {}_xS_{ft}* (1 - {}_xS_{mt})]\}*(1 + IRR)^{-t} \tag{4}$$

which we us to calculate (by method of iteration) the one-period *IRR*s on a postponement decision made by the spouse at each age $x \leq 69$. The survival probabilities used in this study were calculated or obtained from data found in United States Life Tables (2002).

3.2. The data and empirical results

The SSA retirement age benefit adjustment factors, SSB_x/FRB, for beneficiaries born between 1943 and 1954, sorted by age are shown in Column 2 of Table 1; the annual benefit per dollar of FRB differences, $(SSB_{x+1} - SSB_x)$/FRB, are shown in Column 3; and the

Table 1 Retirement age adjustment factors and one-delay credits, retirement credits per $1.00, and actuarial internal rates of return

Initiation age	Retirement benefits as percent of NRA benefits	Credit for each year of benefits postponement after age 62	Increase in future benefits per forfeited dollar if claiming is delayed by one year (λ_x)	Internal rate of return on postponement by one year		
				Single men	Single women	Married couples
(1)	(2)	(3)	(4)	(5)	(6)	(7)
62	75.00%	5%	6.667%	1.80	2.98	3.10
63	80.00	6.67	8.333	3.73	4.92	5.06
64	86.67	6.67	7.693	2.55	3.83	3.97
65	93.33	6.67	7.143	1.44	2.81	2.96
66 (NRA)	100.00	8.00	8.000	2.22	3.66	3.75
67	108.00	8.00	7.407	0.99	2.53	3.69
68	116.00	8.00	6.897	negative	1.45	1.61
69	124.00	8.00	6.452	negative	0.41	1.23
70	132.00	NA	NA	NA	NA	NA

delayed retirement credits per forfeited dollar, $\lambda_x = (SSB_{x+1} - SSB_x)/SSB_x$, are shown in Column 4. The one-period *IRR* for a postponement decision made at each age by single male, single female, and married beneficiaries, respectively, is obtained by substituting λ_x, ${}_xS_{mt}$ and ${}_xS_{ft},$ $x \leq 69$, into Eq. (3) and/or (4), and then by iterative solution. The results for single males, single females, and married couples, respectively are shown in Columns 5, 6, and 7 of the table. The survival probabilities used for calculating the IRRs were calculated using data from United States Life Tables (2002) and U.S. Department of Health And Human Services, Center for Health Statistics (2003).

First note that the annual benefit per dollar of FRB differences reported in Column 3 of the table increases between age 62 and 63, remains flat between ages 63 and 65, increases once again at age 66, and then remains flat. The delayed retirement credits shown in Column 4, by contrast, peak at ages 63 and 66, reaching its highest level at age 63 and second highest at age 66. Each delayed retirement credit, $\lambda_{x,}$ shown in Column 4 of the table, was calculated by taking the ratio of the annual benefit per dollar of FRB difference in Column 3 to the SSA retirement age benefit adjustment factor in Column 2 of the table, and, as simple percentage changes. The λ_x's are unique to this paper only in the narrow sense that, to the best of our knowledge, they have not previously been considered.

A contrast of Columns (3) and (4) brings some rather interesting results into view. Column (3) shows how the level of benefit increase as a result of each one year postponement, whereas Column (4) shows the percentage increase in benefits relative to each annual payment foregone. A retiree who would receive an initial monthly benefit of $1,000 a month at the NRA, for example, would receive an initial monthly benefit of $750 if he or she initiates benefits at age 62. Postponement at age 62 for this retiree, therefore, is tantamount to a one-time investment equivalent of $750 made in exchange for a $50 (or 6.67%) lifetime increase in monthly payment. Similarly, postponing again at age 63 involves a one-time investment equivalent of $800 made in exchange for an additional $66.67 (or 8.33%) lifetime increase in monthly payment. Postponing again at ages 63, 64, or 65, by contrast, result in less attractive tradeoffs–requiring increasing investment equivalents in exchange for declining percentage increases in periodic or monthly payment.

The delayed retirement credits, λ_x, shown in Column 4 of the table suggest that the SSA adjustment factors may not be as fair or consistent as is commonly believed. λ_x peaks at ages 63 and 66, reaching a global maximum at age 63 and a local maximum at age 66; λ_x is otherwise a decreasing function of age. This appears to be a bureaucratic oddity because if the SSA adjustment factors were fair λ_x should be a monotonically increasing function of age.

Turning to Columns 5, 6, and 7 of the table we note that the relative magnitudes of the *IRR* for each demographic group vary with age, and move up or down in lockstep with each other and with λ_x. We see from Eqs. (3) and (4), however, that IRR is a function of both λ_x and one or more cumulative survival probability distributions. It follows therefore, both from the data and by intuition, that the direction and magnitude of each incremental change in λ_x must have far greater impact in determining the direction and relative amounts of change in the magnitudes of *IRR* than small intra cohort group differences survival probabilities that one observes moving from one age x, $x \leq 69$, to the next.

3.3. Discussion

The implicit assumption underlying any market asset investment decision based on the logic of internal rate of return is that each future cash flow resulting from the purchase of the asset will be continuously and instantaneously reinvested in assets belonging to the same risk class with yields equal to or higher than the *IRR*. Logically, moreover, one should never purchase an asset whose yield is strictly less than the rate of return available on an alternative investment opportunity, but when contrasting alternatives we should be careful to avoid, in effect, mixing apples and oranges.

Social Security benefits due on or after initiation are calculated according to formula set by the SSA, periodically adjusted for inflation, and guaranteed by Uncle Sam. In this sense, initiation at any age $x \leq 69$ is tantamount to the purchase of an annuity contract, whose present value may be higher at age x than at age $x+1$ but would be free of both default and inflation risk when purchased at either age. A Social Security annuity contract, therefore, may be viewed as a quasi risk-free asset, whose return may be contrasted with, but is not necessarily equal to, a market determined riskless real rate the of return.

The *IRR's* reported in Table 1 may be variously interpreted depending on one's interest rate assumption, sensitivity to the vagaries of uncertainty, and attitudes towards financial downside risk. The reader is free to adopt a standard different than ours, but we approach the matter from the point of view of a participant who may assume a conservative or aggressive posture in private investment decisions, but has a policy of relative risk aversion when it comes to the reinvestment of Social Security benefits.

For any cohort group or age $x \leq 69$, the *IRR* on a one-period postponement decision calculated at age x may also be interpreted as the minimum investment yield required to justify initiation at age x. However, on the basis of what rate of return standard or investment philosophy are we to decide which of the two alternative actions is optimal? We believe that the current period yield on Treasury Inflation Protected Securities (TIPS) provides a reasonable basis for contrast. As obligations of the United States Government whose par values are periodically adjusted for inflation, TIPS may be regarded as free from default and inflation risk. Though not free of interest rate/market price risk unless held to maturity, TIPS that do not mature during a holder's lifetime are included in the decedent's estate.

At the time of this writing, recent yields on 30-year TIPS have varied in the neighborhood of from 2% to 2.35%. In this paper, therefore, we use 2.35% as the standard against which to judge the optimality of a decision to initiate Social Security benefits. There are, however, important differences between TIPS and Social Security benefits annuities: unlike the later, TIPS can be liquidated at any time, can serve as collateral for debt and can be bequeathed. Thus, it seems reasonable to argue that current market yields on TIPS will typically understate the investment return required to justify postponement.

Before we turn to the discussion of the main results as reported in Columns 5, 6, and 7 of the table, it should be emphasized that the results apply only to people who are no longer working, or whose earned income is below the level subjected to the earned income tax penalty. The results, moreover, are applicable only to people who are in good health and do not need the Social Security benefits to satisfy basic necessities of life. The results indicate that the yield required to justify initiating at any eligible retirement age is lowest for single males and highest for married couples. From this observation, we should infer that, all other things equal, single males should initiate benefits earlier than single females, and that married couples, according to our assumptions–should do so later than others. This implication is consistent with conclusions reached elsewhere, but, of course, all other things are not necessarily equal. By further inspection of the results, we may also infer that a single male might initiate at the earliest possible age except age 63. At 63 the IRR on postponement to age 64 is 3.73%, a real return that will be difficult to beat without taking on more risk. Single females or married couples might initiate at age 68 or beyond. This implication is consistent with conclusions reached elsewhere–where age 60 versus 65 or 66, age 66 versus 70, or age 62 versus 70 contrasts are made.

We see from Columns 5, 6, and 7 of the table that the minimum yield requirement for every cohort group peaks at age 63 rather than at age 62. Given a TIPS current period yield between 2% and 2.35%, moreover, we also observe from the results reported in Column 5 of the table that a single male could initiate benefits at age 62, but not at age 63 or 64, but again age 65 or any age thereafter. This result may be explained in part by noting that the delayed retirement credits, λ_x, at ages 63, 64 and 65 are approximately 125%, 115%, and 107%, respectively, higher than at age 62; λ_x increases again at age 66, but not by enough to cause IRR to rise above the 2.35% riskless rate assumption, and then falls sharply. As previously noted from Eqs. (3) and (4) *IRR* is determined primarily by λ_x, and, therefore, as evidenced by the results reported in Columns 5, 6, and 7 of the table, the relative magnitudes of the *IRR* in each cohort group move up or down in lockstep with each other and with λ_x.

Given a real and risk-free interest rate assumption of 2.35%, the data in Columns 6 and 7 of the table do not provide a basis for easy extension of this argument to single female or married beneficiaries, as members of these cohort groups cannot be advised to initiate benefits until age 68, by which point λ_x decreases monotonically with age. If the *ex ante* riskless interest rate assumption were increased, say to 3.5%, then the same arguments could be repeated virtually verbatim. There is no need or justification to do this, however, as the point has already been made and is quite general, but there is one last point to be noted before turning to conclusions.

Consistent with the findings in other papers, looking across rows at the data in Columns 5, 6, and 7 of the table, suggests, correctly, that single females and married couples should initiate later than single men, and that married couples should initiate later than a single female. Using a cutoff rate of 2.35%, however, both single female and married couples would be advised to initiate benefits at the same age, 68. At first glance, the implication that married couples will not necessarily initiate at a later age than single females would seem at odds with previously published conclusions, but the reader should bear in mind that, in the case of married couples, some very restrictive conditions were imposed on the model.

The expression set out in Eq. (4), used to calculate *IRR* for married couples, reflects a number of limiting conditions that had to be imposed, else the situation would have been too complex to be represented by a single expression. In particular, the model assumes that the partners in any marriage are the same age, and that the spouse's adjusted own benefit is strictly less than 50% of her spouse's. We cannot relax these assumptions in the model, but we can see what the implication might be where, as is typically the case, such limiting conditions do not apply. If a spouse is younger than her spouse, for example, all other things equal, spousal benefits would be delayed or might not come into play at all, but the probability that she would still be alive at any future date, where she might be entitled to a contingent benefit, would also increase. Relaxing just this one condition, therefore, all other things the same, should cause the spouse's initiation yield requirement to increase. The greater the age difference between spouse and spouse, moreover, the higher the resulting yield requirement will likely be.

Relaxing a second implied assumption, as we promised to do above, that if a spouse's adjusted benefit is sufficiently high to disqualify her from receiving spousal benefit while her spouse is alive, it must also be sufficiently high to obviate the advantage of trading her benefit for his after his death; thus, she is treated as a single female. Assume now, all other things equal, that the spouse's adjusted benefit exceeds 50% of her spouse's adjusted benefit, but is nevertheless less than his. In this case it would be to a spouse's advantage to trade her benefits for her spouse's, if she survives him, when he dies, which also cause the spouse's required minimum yield to increase. Relaxing both conditions simultaneously, it seems reasonable to suppose that married men should be advised to initiate benefits later than single females, and perhaps at the latest permissible age.

4. Conclusions

Previously published papers contrast the financial implications of initiation at the earliest retirement age of 62 against postponement until age the NRA or at age 70, which is currently the latest initiation age permitted by law. However, the SSA permits beneficiaries to revisit the problem each month, starting at age 62 until one month before age 70–there is no requirement that a decision should be made at the EEA, presently age 62. Indeed, commensurate with higher life expectancies, fewer beneficiaries voluntarily retire from active employment or initiating benefits at age 62; though approximately 60% currently opt to early initiation of benefits. As employed beneficiaries are now permitted to initiate benefits at the NRA without being subject to an income offset penalty, it behooves employed beneficiaries to calculate the financial implications of initiating benefits at that age.

This paper breaks new ground by supposing, consistent with law, that single or married participant's view an optimizing initiation/postponement decision in terms of a sequential decision processes, rather than in terms of a single evaluation made at the early or normal retirement age, as defined by the SSA. The question asked at each age, according to this sequential approach, is not whether to initiate now or postpone until some time in the distant future, but whether to initiate now or postpone for just one period. This question is asked over and over, according to the model introduced in this paper, until the decision is made to initiate benefits is made or age 70 is reached–whichever comes first. However, at each step along the way, how should one decide?

Logically, one should not purchase any asset whose yield is strictly less than the rate of return available on an alternative investment opportunity, or initiate benefits, therefore, when postponement promises a higher yield. When contrasting yields, however, it is important to remember that while future Social Security benefit cash flow is free of both default and inflation risk, that the yield obtained by reinvestment of social security benefits in risky assets is not risk-free. To avoid, in effect, mixing apples and oranges, current period yield on TIPS serve in this paper as a basis for contrast.

We are pleased to note similarity in many of the conclusions that would be reached in the framework of our model, and results published elsewhere–though based on very different models and assumptions. We should not overstate the importance of such agreement, where it exists, however. The fact that we are using a very conservative market rate of return assumption, based on TIPS, suggests the possibility that there might be less agreement than appears to be the case. In as much as the sequential approach introduced here takes up decisions made between ages 63 and 65, and between 67 and 69, and others do not, significant disagreement may arise, if, for example, one interprets the implications of, say, a 62 versus 66, as also applying at ages 63, 64, or 65.

Our results show that the relative magnitudes of the *IRR* in each cohort group vary with age and move up or down in lockstep with each other and with annual changes in the delayed retirement credit, and that the delayed retirement credit function, though calculated on the basis of an SSA age benefit adjustment factor schedule, is bimodal. Thus, we observed that a beneficiary who might be advised to initiate benefits at one age $x \leq 69$ might not in the next period. This result, which is fully explained above, could not be uncovered using an aggregative model.

It seems appropriate to offer one final caveat before closing: The rates of return on investments here as elsewhere, and/or the postponement decision yields, while obtained by standard method widely regarded as "tried and true," are nevertheless aggregative abstractions of reality. The calculated rates, therefore, which may be appropriate for use by say a financial institution when considering pricing annuity contracts, do not necessarily correspond to the ex post yield that any individual investor or Social Security beneficiary will actually receive. Unlike the SSA, for example, whose average future benefit payouts can be predicted *ex ante* with great accuracy and precision because the averages are calculated over the lifetimes of many beneficiaries, the same law of large numbers does not apply to the individual.

From the point of view of an individual beneficiary death is a personal and unique event–not an impersonal statistic. As benefits are paid only to live beneficiaries, moreover, a Social Security postponement decision may be more akin to participation in a game of chance in which the individual is subject to a variant form of gambler's ruin at death, than a random process that is necessarily ongoing. In the framework of the current literature, this paper, we feel, breaks new ground. Nevertheless, the authors admit to a concern that not enough attention has been paid here to various micro economic aspects of the benefit initiation versus postponement issue, and that there is more work to be done.

Notes

1. Ignoring the possibility of imaginary roots.
2. The use of a higher value, representing some age beyond 100, would, on the other hand, be of little value, which may help explain why the data is not available. Actuarial net present values and internal rates of return are relatively insensitive to cash flows beyond age 90, because the survival probabilities used to weight such cash flows are relatively low while the exponent in any term $(1 + \mathrm{IRR})^{-t}$ is relatively high.
3. Multiplying through in Eq. (2) below by SSB_x, and substituting variable definitions from Eq. (1), we get:

$$\begin{aligned} SSB_x &= SSB_x * [(SSB_{x+1} - SSB_x)\ /SSB_x]*\Sigma_{t=x+1}(1 + \mathrm{IRR})^{-t} \\ &= (SSB_{x+1} - SSB_x) * \Sigma_{t=x+1}(1 + \mathrm{IRR})^{-t} \end{aligned}$$

4. Please see Footnote 2.

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