Analyzing 400m Running Performance Based On: Track Geometry, Air Resistance, and Altitude

By

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ABSTRACT

Analyzing 400 m Running Performance Based On: Track Geometry, Air Resistance, and Altitude

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Previous ordinary differential equation (ODE) models by Keller, Quinn, and Alday and Frantz have analyzed the effects of air resistance, altitude and track geometry on sprinting performance. The majority of past work has focused on these effects in the 100m and 200m running events, however, only limited research has been done for the 400m. The purpose of this paper is to continue expanding upon these previous models for the 400m in hopes to apply them to varying tracks geometries, and then to see how air resistance and altitude effects these track geometries. We first simulated the impact that the standard track has on performance for windless conditions, with air resistance, and with altitude and compared our results to those of the previous models. Then we modified the standard track model to simulate the double-bend track for each of the three conditions. MATLAB® was used in order to solve the ODEs associated with analyzing 400m race times.
CHAPTER 1
INTRODUCTION

1.1. BACKGROUND AND OBJECTIVE

Track and field, also known as athletics in many countries across the globe, ranks as the eighth most popular sport worldwide, as it draws attention from countries from all religions, ethnicities, genders, and socioeconomic statuses. (Brown) Track and field has remained a prominent sport throughout its existence starting as one of the original sports comprising the ancient Olympic Games to becoming today’s most watched sport at the world’s most popular event the modern Olympic Games. (Brown)(IOC, 2016) Many events within athletics are highly sensationalized, specifically the 100m and 4x100m relay\(^1\), and much research has been completed on analyzing race tactics and performance within these events.

Previous mathematical models used to simulate race performance by Hill, Keller, Quinn, and Ward-Smith and Radford have utilized systems of Ordinary Differential Equations (ODEs) to model the effects of track geometry, air resistance and altitude on the 100m, 200m, and 4x100m races. (Hill, 1928)(Keller, 1974)(Quinn, 2003)(Ward-Smith, 2002) However, not until recently has the 400m been modeled in this manner. Some of Quinn’s later works and Alday and Frantz’s work were able to expand upon some of these earlier models in order to simulate the 400m. Creating a model for this event proves to be a more complex task than the 100m and 200m, as it comprises two bends and two straights. The project in the ensuing paper intends to further investigate the effects on the 400m for various track geometries, while considering air resistance and altitude.

\(^1\)In the 4x100m relay four athletes each complete 100m of the total 400m.
1.2. THE IMPORTANCE OF ANALYZING THE 400m RACE

Further research needs to be completed for modeling the 400m race, because as stated previously this sprint event has not been analyzed or modeled to the extent that its counterparts, the 100m and 200m have been. This could be for several reasons: the 100m and 200m are more sensationalized by the media, there are more name recognition stars within these events, wind effects are viewed as more of a factor within race performance, and simply the 400m being a harder race to model.

However, the need to replicate the 400m event extends past the little previous research that has been conducted. The results of Alday and Frantz’s model suggest that track geometry, air resistance, and altitude may have some serious implications on a given 400m race, implications that were once thought to only exist for the shorter sprint events. (Alday, 2010) These three factors can impact whether a world record (WR) is set, whether one runner gains an advantage over another, which in turn can influence who wins a race or Olympic Gold.

For instance, even though every athlete runs exactly 400m around the track for the 400m event, regardless of what lane they are assigned, each lane has a different radii. Thus, this difference in radii causes some runners to run more or less of the bend at a crucial point in the race. This point being when the runners are trying to reach maximum velocity, usually the first 50-100m of the race.2 (Alday, 2008) With all else being equal, this can provide an advantage to the runners that get to run more distance on a straightaway while they are trying to reach maximum velocity, rather than trying to accomplish this around a tight bend.

Air resistance can also provide an advantage or disadvantage to a runner in a 400m race. This is an obvious fact for the 100m because the race is run in a straight line with the wind hitting the runner

---

2 It should be noted that every athlete will run the same cumulative distance for the bends and straights, respectively. In order, to make it so every runner races 400m, they are staggered at the starting line, within their respective lane. Thus, some athletes will run the same distance around both bends (lane 1), while the rest will run a shorter first bend and longer second bend depending on their stagger.
from the same direction throughout the whole race. In the 100m, an athlete is not going to have to battle a tailwind and a headwind in the same race. However, intuitively it would seem that since the 400m is a closed circuit the headwinds and tailwinds would cancel themselves out, but this is not the case. (Alday, 2010) The wind speed and direction can also affect runners within the same race differently. This is due to the angle or direction of the wind in relation with the radii of the curve that a specific runner has to traverse.

Though wind speed is currently regarded for short sprint events like the 100m and 200m when allowing records to be ratified, track geometries, wind direction, and altitude\(^3\) are not. Interestingly enough, wind speed is not considered for record setting purposes in the 400m. This poses a huge problem, when these factors can have a great impact on the outcome of a race, providing advantages or disadvantages to some or all athletes. Since only wind speed and not wind direction is taken into account for record setting purposes in certain events, some times that have been run over the allowable wind speed, \(2 \frac{m}{s}\), have not counted as a record, even though the direction of the wind in reality provided a disadvantage to that runner. Also, with all else being equal, it has been shown in previous studies that lane 8 is the fastest lane, due to it having a wider radius and having the least amount of distance to run on the first bend while reaching optimal speed. On the contrary, lane 1 is the slowest lane due to it having a tight radius and having the greatest distance to run on the first bend. Another advantage to runners is from an increase in altitude because the air density is lower than at sea level. The bottom line is that more factors other than wind speed need to be included when determining records and the “fairness” of a race. Then further, all these factors need to be considered for the 400m event. It has been shown that wind direction, track geometry, and altitude all have an effect on the outcome of a race, and thus, should be acknowledged. The study that this paper will conduct hopes to extrapolate upon these previous findings.

\(^3\)Times that are run above an altitude of 1,000m are marked with an ‘A,’ to show it was altitude assisted. Though there is no rule for record setting purposes. (Alday, 2010)
2.1. EARLY MODELS

The sport of track and field has been around for centuries, however, the attempt to mathematically model certain aspects of the sport did not begin until the twentieth century. During this time it became of particular interest to Archibald V. Hill, a British physiologist and biophysicist to model the effects of wind on sprinting performance, an idea that has pioneered much further research in the field. (Alday, 2010) In order, to complete his model, Hill employed Newton’s Law for the energy balance of a runner. (Alvarez-Ramirez, 2002) Though, Hill’s work went unpublished, Joseph B. Keller, was still able to obtain the previous model and expand upon it. Keller, a professor of Mathematics and Mechanical Engineering at Stanford University turned the basic ideas of Hill into an optimal control problem, which he solved via calculus of variations. (Alvarez-Ramirez, 2002) He then went on to create the most basic equation of the motion of a runner, which can be seen as follows:

\[
\frac{dv(t)}{dt} = f(t) - \frac{1}{\tau}v
\]

where \( v(t) \) is the runner’s velocity at time \( t \), in the direction of motion and \( f(t) \) is the runner’s propulsive force\(^4\) per unit mass. The resistive force\(^5\) is linear in velocity \( v \), and \( \tau \) is the time constant. (Quinn, 2003)

Due to the link between the work of Hill and Keller, this model later became known as the Hill-Keller model.

Though the Hill-Keller model did an adequate job of modeling the motion of a runner, it has had its fair share of criticisms. One specific objection lies within one of the model’s key assumptions, that a runner can maintain a certain pace, indefinitely without decay. However, it is impossible, physiologically, for a runner to keep a constant pace forever, because eventually the muscles will fatigue and the limited amount of energy stored in the muscles will become depleted throughout the course of the exercise.

\(^4\) Propulsive force is the force on a body or object used to accelerate it forward. ("Propulsive Force", 2016)
\(^5\) Resistive force is the force whose direction is opposite to the velocity of the body. ("Resistive Force", 2016)
Future designs will transform the Hill-Keller model into ones that will take into account the decay of the runner’s propulsive force.

2.2. QUINN’S MODELS

Mike D. Quinn of Sheffield Hallam University has been at the forefront of the research completed for analyzing the effects of wind, altitude, and track geometry for sprint events. He has published three papers on the topic, the first two modeled the effects of wind and altitude for the 200m (Quinn, 2003) and the 400m (Quinn, 2004) and the third simulated track geometry for both the 200m and 400m (Quinn, 2009). Each of these three papers has built upon the other, thus coming to a more complete mathematical model of sprint racing.

2.2.1 QUINN’S IMPROVEMENTS OF THE HILL-KELLER MODEL

His first work, completed in 2003, expounded upon Keller’s model to show the effects of wind and altitude on the 200m by considering the reaction time of the runner, as well as, air resistance, which is dependent on the relative velocity between the sprinter and the air. (Quinn, 2004) Thus, Quinn’s improved model became:

\[
\frac{dv(t)}{dt} = f(t) - \frac{1}{\tau}v - \alpha(v - v_w)^2
\]  

(2)

where \(v_w\) is the velocity of the wind relative to the ground and

\[
\alpha = \frac{\rho C_d A}{2M}.
\]  

(3)

For \(\alpha\), the variables are defined as: \(\rho\) is the air density, \(C_d\) is the coefficient of the drag, \(A\) is the frontal area of the athlete, and \(M\) is the mass of the athlete. (Quinn, 2003)

The results Quinn arrived at after utilizing this model for the 200m were that there is an evident combined effect of wind and altitude⁶ on the outcome of the race. The effect was so apparent that when correcting performances for the two factors on the all-time top five times lists for men and women there was significant movement in people’s position on the lists. (Quinn, 2003) The model also showed that

⁶ Altitude is taken into account through the air density \(\rho\), which is reduced at an increase in altitude.
wind direction along with wind speed impacts finishing times run for the 200m. An impact where two races run with the same wind speed reading but differing wind directions can produce times that vary by as much as half a second. (Quinn, 2003) In a race as short as the 200m, half a second can be the difference between running an average time and setting a WR. Thus, as mentioned in the introduction it is imperative that wind direction be recognized along with wind speed when determining wind-legal times, because there is an obvious effect. Though this model improves Keller’s original equation, Quinn’s 2003 formula still does not address or fix the assumption presented in the Hill-Keller model that a runner can maintain the same pace indefinitely. (Quinn, 2003)

2.2.2 QUINN’S 400m MODEL

Quinn then went on to publish his second work in 2004 which extended his previous work to the 400m event, as well as, fixed the velocity profile issue that had plagued many prior models. Through other studies analyzing sprint performance, it has been shown that world-class sprinters reach their maximum velocity well before the finish line. For example, in the 200m and 400m, athletes reach their top speed between the first 50 and 100m, before decreasing to the finish line due to muscle fatigue. (Quinn, 2003) Thus, Quinn extended his own 2003 (Quinn, 2004) model to acknowledge this phenomenon, where the resulting formula became:

\[
\frac{dv(t)}{dt} = F e^{-\beta t} - \frac{1}{\tau}v - \alpha(v - v_w)^2 \tag{4}
\]

Here we have that \( v = \frac{ds}{dt} \), where \( s(t) \) is the distance traveled. Then, \( Fe^{-\beta t} \) is substituted in for \( f(t) \) as the new term for the runner’s propulsive force per unit mass to reflect the fatigue of the athlete. In this term, \( F \) is defined to be the maximal force an athlete can produce, thus \( f(t) \leq F \), \( \beta \) is the decay rate, and \( t \) is time. (Quinn, 2004)

After implementing this decay rate term, Quinn wanted to create a way to replicate the velocity profile of an athlete running around the bends. This idea proves to be very important considering in the 400m race on an IAAF standard track, 58% of the race is run around the bends, because the straights are
only 84.4m and the bends 115.6m. (Quinn, 2004) The only physical difference between running on a straight and running on a curve is the centrifugal forces on the sprinter. (Mureika, 1997) Thus, while a sprinter traverses a bend, they must lean into it generating a lateral foot force that balances the centrifugal acceleration, which then decreases the maximum speed that the sprinter can reach around the bend. Assuming that there is a limit to the force that is produced at a sprinter’s maximum velocity, the vertical component of leg force is thus decreased on the bends. In an attempt to compensate for this, the runner’s foot contact-time increases, resulting in a reduced speed. (Quinn, 2009) Intuitively this makes sense, since a sprinter’s speed around a bend will be less than when they are running in a straight line. (Quinn, 2003) The resulting reduced speed is dependent on the athlete’s straight line speed and the radius of the lane’s bend. (Quinn, 2004) To correct the athlete’s velocity for the effect of lane radius, Quinn assumed that the sprinter’s straight line speed is \( v_0 \), which is then reduced to \( v \) when the athlete runs around a bend with radius, \( r \). (Quinn, 2004) Quinn then defined the non-dimensional variables \( \omega = \left( \frac{v}{v_0} \right)^2 \) and \( \lambda = \frac{r}{v_0^2} \), where \( g \) is the gravitational acceleration, \( \lambda \) is the dimensionless parameter called a reciprocal Froude number, which allows us to compare the velocities of runners with differing peak velocities, as well as, to compare both gravitational and inertial forces. (Alay, 2008) Then the relationship between \( \omega \) and \( \lambda \) is expressed through the following equation (Alay, 2008):

\[
\omega = \left( \frac{v^2}{2} + \sqrt{\frac{v^4}{4} + \frac{g^2}{2g}} \right)^{\frac{1}{2}} + \left( \frac{v^2}{2} - \sqrt{\frac{v^4}{4} + \frac{g^2}{2g}} \right)^{\frac{1}{2}}
\]  

(5)

The velocity of the runner around the bend is then calculated by solving (4) and \( v = \frac{dv}{dt} \) using the correction \( v = v_0 \sqrt{\omega} \). (Quinn, 2004) This method is used until the sprinter enters the straight. The value of the wind velocity, \( v_w \), depends on the wind direction; the actual wind velocity, \( u_w \); the distance traveled,

---

7 Centrifugal force is the apparent force that draws a rotating body away from the center of rotation, caused by the inertia of the body. (Lucas, 2015)
8 Centrifugal acceleration is the rate of change of the velocity of a moving body with respect to the given time on a curved path. [41]
9 Froude numbers are a dimensionless parameter used to provide the ability to compare resistances of objects with differing sizes. (Alay, 2008)
and the radius of the specific lane, $r$. Thus, $v_w$ will vary continuously around the bend even when the winds are of constant velocity, where a positive $v_w$ will correspond to a tailwind and a negative $v_w$ to a headwind. (Quinn, 2004) Quinn then divided the 400m track into four sections: the first bend, the back straight, the second bend, and the finishing straight. Then letting the actual wind speed be $u_w(t)$ blowing at an angle $\theta$ to the finishing straight, as shown in Figure 1, the wind facing the sprinter in each lane of the track could be calculated. Thus, the wind velocity for an athlete at each of the four quadrants of the track is:

First Bend: 
$$v_w = u_w \cos(\theta + \frac{231.2-s}{r}) \quad (0 \leq s \leq 231.2 - \pi r)$$  

Back Straight: 
$$v_w = -u_w \cos(\theta) \quad (231.2 - \pi r \leq s \leq 315.6 - \pi r)$$

Second Bend: 
$$v_w = u_w \cos(\theta + \frac{315.6-s}{r}) \quad (315.6 - \pi r \leq s \leq 315.6)$$

Finishing Straight: 
$$v_w = u_w \cos(\theta) \quad (315.6 \leq s \leq 400)$$

(Quinn, 2004)

Figure 1: Wind Direction Diagram. This figure shows how wind direction is measured in a 400m race. (Quinn, 2004)

Quinn then went on to take into account how altitude affects running performance, due to the reduced air density and accompanying reduction in the partial pressure of oxygen.\(^{10}\) Thus, he modeled the air density, $\rho_H$, at an altitude of $H$ meters above sea level related to the air density at sea level, $\rho_0$, by the equation:

---

\(^{10}\) Partial pressure of oxygen is the measurement of oxygen in the arterial blood. (Leader, 2016)
\[ \rho_H = \rho_0 e^{\frac{-gH}{RT}} \]  

(10)

In equation (10) \( T \) is the air temperature in Celsius, \( g \) is gravitational acceleration and \( R = 287 \, J \, kg^{-1} \, K^{-1} \) is the gas constant. Since the drag coefficient, \( C_d \), is not thought to be significantly affected by the increase in altitude and any changes in gravitational acceleration due to location are not significant, it seems that the advantage gained from running at altitude is only from the decrease in air density. (Quinn, 2004)

As altitude increases, the partial pressure of the oxygen drops, resulting in a decrease in the percentage of oxygen saturation in the blood. Thus, there is a decrease in the amount of oxygen supplied to the muscles from the aerobic energy system.\(^{11}\) (Quinn, 2004) For altitudes less than 3,000m, the amount of reduction in the oxygen saturation is very small, but can still have negative effects on an athlete’s performance.\(^{12}\) However, the effect is dependent on the altitude and the contributions of the aerobic and anaerobic energy systems,\(^{13}\) respectively. (Quinn, 2004) For the 400m as much as 40% of the energy used is from the aerobic energy system, with this percentage being slightly lower for world-class runners. (Quinn, 2004) Thus, it makes sense that when at altitude the propulsive force, \( F e^{-\beta t} \) declines at a faster rate than when at sea level. Quinn was able to formulate this through the next equation:

\[ \beta = \beta_0 (1 + \gamma \sigma H) \]  

(11)

In equation (11) the propulsive force term \( \beta \) now depends on the altitude, \( H \), the relative contribution of the aerobic energy system, \( \gamma \), and \( \beta_0 \), which is the parameter value at sea level. Quinn also assumed that the aerobic contribution of \( \beta \) varied with altitude at a rate proportional to the decrease in oxygen saturation in the blood, \( \sigma \). (Quinn, 2004)

---

\(^{11}\) The aerobic energy system is the means of production of energy through processes that require oxygen. (PT Direct, 2016)

\(^{12}\) The effects of a reduction in the oxygen saturation at high altitudes, severely impacts distances runner, who rely mainly on the aerobic energy system throughout their race. (Quinn, 2004)

\(^{13}\) The anaerobic energy system is the means of production of energy through processes that do not require oxygen. (PT Direct, 2016)
Quinn, then used fitted data parameters for windless conditions in lane 4 to solve numerically, with a fourth order Runge-Kutta method, his improved system of non-linear equations of motion. (Quinn, 2004) The results that he produced were surprising. His simulations showed that with certain wind speeds and directions, an advantage could potentially arise in terms of both finishing times and velocity profiles for runners in certain lanes over running in windless conditions. (Quinn, 2004)

2.2.3. QUINN’S FINAL MODEL

Quinn’s final model set out to mimic the effects of various track geometries in windless conditions for the 200 and 400m sprint events. Prior to running the simulations he proposed that there exists an optimum track geometry that, with all else being equal, would produce the fastest times for all lanes. (Quinn, 2009) Using the same model from his most recent paper (Quinn, 2004), he was able to determine the effect bend curvature has on 400m running performance.

In his recent work, Quinn analyzed two types of tracks: those with semi-circular bends and those with a double-curve bend. The track type with semi-circular bends is the simplest track design as it consists of two parallel straights and two semi-circular bends with equal radii. For this study Quinn modeled three semi-circular bend tracks: the IAAF standard track, one he called the “minimum radius” track, and the other he called the “maximum radius” track. The track type with double-curve bends is more complicated because each bend is made up of circular arcs of two differing radii. (Quinn, 2009) For this study Quinn modeled three double-curve bend tracks, which he called “double-curve 1,” “double-curve 2,” and “double-curve 3.” More in depth parameters and specifications for certain types of tracks will be discussed later.

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14 The double-curve bend track is also known as the double bend track.

15 The specifications for a semi-circular track states that the radius must be between 35 and 38m. The IAAF standard track has a radius of 36.5m, bends of length 115.6 m, and straights of 84.4m. Whereas the “minimum radius” track is measured on the minimum radius value of 35m, and the “maximum radius” track is measured on the maximum radius value of 38m. (Quinn, 2009)

16 The “double-curve 1” track has the largest radii, shortest straights, and longest bends of the three double-curve bend tracks, with the “double-curve 2” having a smaller radii, longer straights, and shorter bends. Then the “double-curve 3” track having the shortest radii, longest straights, and shortest bends. (Quinn, 2009)
The results of Quinn’s 400m track geometry study showed that, with all else being equal, lane 1 is the slowest, and each subsequent lane gets faster than the previous one. (Quinn, 2009) Intuitively it would seem that the tracks with the widest bends would be the fastest and the tracks with the tightest bends would be the slowest. However, in Quinn’s simulations this was not necessarily the case. This is because more factors contribute to a sprinter running fast than just track radius and geometry, specifically, the velocity profile of the runner and the amount of time they spend running in each section of the track. Though in general the tracks with the most optimal design were those with the larger radii, such as the standard track, the “maximum radius” track and the “double-curve 1” track, and the worst tracks were the “double-curve 2” and “double-curve 3” tracks, which had smaller radii. (Quinn, 2009)

Therefore, Quinn has set the precedent for much research in analyzing the effects of track geometry, air resistance, and altitude on the 400m event. With Quinn’s improvements of the Hill-Keller model and his own formulations, Quinn has created a foundation for further investigation.

2.3. ALDAY AND FRANTZ’S CONTINUATION OF QUINN’S WORK

Finally, Alday and Frantz set out to validate and expand upon Quinn’s second paper on the effects of wind speed and altitude on the 400m event. (Quinn, 2004) They used all of Quinn’s ODEs and parameter values, and then used a program in Maple to run their simulations for the IAAF standard track, the equal quadrant track,\(^\text{17}\) and the Ancient Greek Olympiad track.\(^\text{18}\)

First, Alday and Frantz started with corroborating Quinn’s simulations and results for lane 4 of the IAAF standard track in windless conditions. Their results ended up being very similar to Quinn’s, with both their finishing times being exactly the same, 43.18s. (Alday, 2010) Then, they took into account wind speed of 2 m/s for varying angles of the track. From this simulation, they noticed that for each lane

\(^{17}\) The equal quadrant track is composed of two straights of 100m and two bends of 100m. The longer straights and tighter radii of the equal quadrant track contribute to why athletes tend to run slower on this type of track versus the IAAF standard track, as will be seen later in the paper.

\(^{18}\) The Ancient Greek Olympiad Track consists of two bends of 20m with radius 6.37m and two straights of 180m. (Alday, 2010) It should be noted that this type of track is no longer in use, its purpose for being analyzed is to see the effects of track geometry in general.
there are two significant directions, with approximately opposite compass points where there is a definite advantage or disadvantage if the wind blows in one of these directions. (Alday, 2010)

Since Alday and Frantz were able to reach, in general, equivalent results as Quinn, they were then ready to implement their model to simulate other types of tracks, specifically the equal quadrant track and the Ancient Greek Olympiad track. In order to model these designs, however, they had to change the parameters related to the strength of the relative wind speed as a function of the distance around the track. (Alday, 2010) For example, to model the equal quadrant track they had to change equations (6),(7),(8), and (9) for the relative wind velocity, \( v_w \), on different portions of the track to ones that would reflect its 100m straights and 100m. These new equations are as follows:

First Bend: 
\[
v_w = u_w \cos(\theta) + \left(\frac{200 - s}{r}\right) (0 \leq s \leq 200 - \pi r)
\]

(12)

Back Straight: 
\[
v_w = -u_w \cos(\theta) (200 - \pi r \leq s \leq 300 - \pi r)
\]

(13)

Second Bend: 
\[
v_w = u_w \cos(\theta + \frac{(300 - s)}{r}) (300 - \pi r \leq s \leq 300)
\]

(14)

Finishing Straight: 
\[
v_w = u_w \cos(\theta) (300 \leq s \leq 400)
\]

(15) (Alday, 2010)

The equations (12), (13), (14), and (15) are then transformed in a similar fashion in order to reach the appropriate equations for the Ancient Greek Olympiad track, which consists of two 180m straights and two 20m bends. (Alday, 2010)

Now that Alday and Frantz had the right equations to model the equal quadrant track and the Ancient Greek Olympiad track, they proceeded to run their simulations and ultimately compare these results to those of the IAAF standard track. While comparing the IAAF standard track to the equal quadrant track under windless conditions, the finishing times were 43.18s and 43.24s for the two tracks, respectively. Thus, the IAAF standard track was found to be 0.06 seconds faster than the equal quadrant track, which is explained by its larger lane radii. (Alday, 2010) Overall, for the IAAF standard track and the equal quadrant track, the largest disadvantage is to the runner in lane 1 during windless conditions, due to the tighter bends forcing the runner to spend less time in the air and more time on the ground. This
effect is more pronounced for the equal quadrant track, as its radius for lane 1 is even tighter than that of the IAAF standard track. (Alday, 2010)

Then comparing the finishing time of lane 4 in windless conditions for the Ancient Greek Olympiad track to the IAAF standard track, Alday and Frantz found that the former is slower than the latter by 0.25s, with the Ancient Greek Olympiad track’s time being 43.43s. (Alday, 2010) This is once again due to the tighter turns of the Ancient Greek Olympiad track. Though the differentials of 0.06s and 0.25s between tracks may seem miniscule, in terms of a sprint event this is a long time. Considering that most runners finish within tenths of seconds of each other, this could mean the difference between setting a WR, Personal Best (PB) or just running an average time.

After comparing for windless conditions, Alday and Frantz took into account a wind speed of $2 \text{ m/s}$ at varying angles of the track. For the equal quadrant track the results were similar to those that were arrived at for the IAAF standard track, in that a constant wind of $2 \text{ m/s}$ provided advantages and disadvantages for each lane depending on the direction and angle that the wind was blowing. (Alday, 2010) Then it was found that for the Ancient Greek Olympiad track reducing the headwind on the straights would provide the most advantage to the runner. For all three tracks the greatest advantage is provided to the sprinter when they are facing a wind direction of $240^\circ$ and the most disadvantage when facing a wind of $30^\circ$. (Alday, 2010) This can be interpreted to mean that if a wind has to be encountered, especially on the bends where the most energy is required, then more of an advantage is afforded when the runner faces a tailwind earlier in the race and a headwind later on. Thus, a headwind earlier in the race has a greater effect on reducing an athlete’s energy for the rest of the race, as opposed to the benefits that are achieved from a tailwind at the beginning. (Alday, 2010)

When comparing for the effect of altitude for each of the three types of tracks, the results were the same across the board: the higher the altitude the faster the times. For an altitude of 500m on the equal quadrant track the average time correction was -0.09s, whereas, for an altitude of 2500m the average time
correction was as much as -0.42s. (Alday, 2010) With all else being equal, that is a time correction of a little less than half a second, which is only attributed to an increase in altitude. However, unlike with the effects for wind conditions, the effects for altitude will interact with every runner in every lane equally, thus not providing an advantage or disadvantage to any given runner.

Thus, Alday and Frantz have shown that factors outside of the fitness ability of an athlete, their strategy, and their mental toughness go into that athlete setting a PB, WR, or winning a medal. Sometimes factors outside the athlete’s control, such as lane assignment, wind conditions, and the altitude of the race, can positively or negatively impact their race performance.

The works of Quinn and Alday and Frantz are what has inspired this current project. The goal is to expand upon these studies, making them even more accurate and applicable to more types of track geometries.

CHAPTER 3
RESEARCH DESIGN AND METHODS

3.1. A NOTE ON TRACK GEOMETRIES

Though every outdoor track is a closed circuit and is 400m around, not every track has the same lengths for the straights and bends. There are several different track dimensions that are used for athletics competitions. As discussed in the Introduction and the Literature Review, each of these differing track designs can have an impact on the outcome of a race, in terms of both finishing time and placing. For all tracks the width of each lane is 1.22m ± 0.01m. The lane width is measured radially from the inside edge of the lane to the inside edge of the adjacent lane. Lane 1 is measured starting 0.30m from the outer edge of the inside kerb,19 while all other lanes are measured starting 0.20m from the outside edge of the inside lane marking. (Quinn, 2009) The reason the measurements are taken this far into the lane, is to account

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19 The kerb is the buffer between the track and the infield. Usually, the kerb is marked with a raised rail or simply a white line.
for where the athlete is running within their respective lane. This section intends to give a better understanding of each of the main track geometries that will be used throughout this study.

3.1.1. IAAF STANDARD TRACK

The IAAF standard track typically has lane 1 bends of length 115.6m with lane 1 radius of 36.8m and straights of 84.4m. (Alday, 2010) Though these measurements are typical for this type of track, the IAAF allows for a minimum radius of 35m and maximum radius of 38m. The former results in a track consisting of the tightest allowable bends and longest straights of 89.10m, while the latter the widest bends and shortest straights of 79.68m. (Quinn, 2009) The lane radii for the typical standard track are what was used in our model, and can be seen in the following table, Table 1.

<table>
<thead>
<tr>
<th>Lane</th>
<th>Radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.80</td>
</tr>
<tr>
<td>2</td>
<td>37.92</td>
</tr>
<tr>
<td>3</td>
<td>39.14</td>
</tr>
<tr>
<td>4</td>
<td>40.36</td>
</tr>
<tr>
<td>5</td>
<td>41.58</td>
</tr>
<tr>
<td>6</td>
<td>42.80</td>
</tr>
<tr>
<td>7</td>
<td>44.02</td>
</tr>
<tr>
<td>8</td>
<td>45.24</td>
</tr>
</tbody>
</table>

Table 1: Standard Track Lane Radii. This table displays the radius for each lane of the typical standard track. (Alday, 2008)

The IAAF standard track is popular for multiple reasons. The wider bends enhance a runner’s performance, lessen the risk for injury, and allow the ability to accommodate soccer and football fields, within the track. (Alday, 2010) A diagram of the standard track can be seen in Figure 2 as follows:
Figure 2: IAAF Standard Track Dimensions. This track has lane 1 bends of length 115.6m with kerb radius of 36.5m, and straights of length 84.4 m. It should be noted that to calculate lane 1 radius, 0.3m is added to the kerb radius. (Meinel, 2008)

3.1.2. DOUBLE-BEND TRACK

Another type of track that is allowed by the IAAF is the double-bend track. While the standard track composed of two straights and two bends with equal radii, the double-bend track is made up of two straights and two bends each with two different radii. In the double-bend track the radius at the start and finish of the curve is different from the radius of the middle part of the curve. (Quinn, 2009) Figure 3 details how the radii of a double-bend track are oriented.

Figure 3: Double-Bend Curve Radii Diagram. This diagram shows the geometry of the double-bend track, where the middle section of the curve is an arc of angle $Q$ with radius $R$, and the rest of the curve is made up of two equal arcs of angle $q$ and radius $r$. (Quinn, 2009)
The IAAF allows for three different geometry types for the double-bend track, each with differing inner radius, \( R \), and outer radius, \( r \). The first type, “double-curve 1” has \( R=51.54\text{m}, r=34.00\text{m} \), bends of 120.01m, and straights of 80.00m, the second, “double-curve 2” has \( R=48.00\text{m}, r=24.00\text{m} \), bends of 101.48m, and straights of 98.52m, and finally, the “double-curve 3” has \( R=40.02\text{m}, r=27.08\text{m} \), bends of 102.74m, and straights of 97.26m. (Quinn, 2009)

On the next page, Figure 4 outlines the geometry for the “double-curve 1” track, and is the type of double-bend track that will be analyzed in our model.

![Figure 4: “Double-Curve 1” Track Dimensions. This diagram shows how a double-bend track is measured. For “double-curve 1” tracks \( R=51.54\text{m}, r=34.00\text{m} \), bends equal to 120.01m, and straights equal to 80.00m. (Meinel, 2008)](image)

Then Table 2, outlines the lane inner, outer, and average radii for Quinn’s “double-curve 1” track, henceforth referred to as the double-bend track in our model.
<table>
<thead>
<tr>
<th>Lane</th>
<th>Outer Radius (m)</th>
<th>Inner Radius (m)</th>
<th>Average Radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.3</td>
<td>51.84</td>
<td>38.20</td>
</tr>
<tr>
<td>2</td>
<td>35.42</td>
<td>52.96</td>
<td>39.32</td>
</tr>
<tr>
<td>3</td>
<td>36.64</td>
<td>54.18</td>
<td>40.54</td>
</tr>
<tr>
<td>4</td>
<td>37.86</td>
<td>55.40</td>
<td>41.76</td>
</tr>
<tr>
<td>5</td>
<td>39.08</td>
<td>56.62</td>
<td>42.98</td>
</tr>
<tr>
<td>6</td>
<td>40.30</td>
<td>57.84</td>
<td>44.20</td>
</tr>
<tr>
<td>7</td>
<td>41.52</td>
<td>59.06</td>
<td>45.42</td>
</tr>
<tr>
<td>8</td>
<td>42.74</td>
<td>60.28</td>
<td>46.64</td>
</tr>
</tbody>
</table>

Table 2: Double-Bend Track Lane Radii. This table displays the radius for each lane of the double-bend track.

The main reason for the usage of the double-bend track is for fitting sports’ competition fields within the track. The dimensions of the double-bend track make it easier to put a football, soccer, or rugby field on the infield, as well as, being more cost efficient by being able to build one stadium to house a wide array of sporting events. (Meinel, 2008)

3.2. EULER’S METHOD

Even though previous research on this topic has used a fourth order Runge Kutta method, we decided to use an explicit Euler method. This is because when we used the Runge Kutta method our results were not consistent with those from previous research. However, when implementing the Euler method we were able to arrive at the proper results, as described by Quinn and Alday and Frantz. (Alday, 2008)

Euler’s Method is a procedure used for solving ODEs by generating numerical solutions to an initial value problem of the form:

\[ y' = f(x, y) \]  \hspace{1cm} (16)

\[ y(x_0) = y_0 \]  \hspace{1cm} (17)
Next, we decide what interval, starting at the initial condition that we want to find the solution. This interval is then divided into small subdivisions of length $h$. Then, starting at the initial condition, we generate the rest of the solution by utilizing the iterative formulas:

$$x_{n+1} = x_n + h \quad (18)$$

$$y_{n+1} = y_n + hf(x_n, y_n) \quad (19)$$

We use these iterative equations to find the coordinates of the points in our numerical solution. Finally, we stop the iterative process when we have reached the right end of the intended interval. (Barker, 2009)

One issue with Euler’s method is that it is not very accurate, especially with large step sizes, $h$. Depending on the ODEs presented, a way to fix this issue is by simply decreasing the size of the steps. (Barker, 2009) As, will be seen in the coming sections, this is exactly what we did, and since our solutions matched those of Alday and Frantz, we considered Euler’s Method to be an appropriate procedure for our set of ODEs.

3.3. METHODS USED TO IMPLEMENT PREVIOUS MODELS INTO MATLAB

Several steps have been taken thus far in the project process in order to apply and improve the models used in previous literature. The first step was implementing the Alday model in MATLAB. In order to ensure that our model was running properly, we compared the two models. The final steps then include using the model for various track dimensions not extensively studied, investigating and improving some of the parameter values in the model, and carrying out a case study of the men’s 2016 Rio Olympic 400m final.

3.3.1. MODELING STANDARD TRACK IN WINDLESS CONDITIONS

The first step that was taken to research the effects of track geometry, altitude, and air resistance on the 400m event was to implement Alday’s model for the IAAF standard track into MATLAB. (Alday, 2010)(Quinn, 2004) The equations (4)-(9) were first solved in MATLAB for windless conditions, to make
sure that the model was properly utilized. At first MATLAB’s standard ODE solver ode45\textsuperscript{20} was used to solve the differential equations, however, the results arrived at were very inconsistent. Instead, an explicit Euler Method with small step size was employed, which provided accurate and consistent results when compared to Alday and Frantz’s model. The following parameter values were used when solving the ODEs for men in windless conditions: \( F = 7.91 \, \frac{m}{s^2} \), \( \beta = 0.006 \, s^{-1} \), \( \tau = 1.45s \), \( u = 0 \), \( \theta = 0 \), \( a = 0.00284 \), and the lane radii from Table 1. (Alday, 2010) \( F \), \( \beta \), and \( \tau \) were all fitted by Quinn from data collected from digital high-speed video cameras that recorded athletes during each 50m around the track. (Alday, 2008) The reason both \( u \) and \( \theta \) are 0, is because this simulation does not take into account the effects of wind.

The value for \( \alpha \) was then found by substituting the following parameter values, \( \rho = 1.184 \, \frac{kg}{m^3} \) at 25\(^\circ\)C, a likely temperature for sports races, \( C_d = 0.715 \), \( A = 0.51 \, m^2 \), \( h = 1.85 \, m \), \( M = 76 \, kg \), into equation (3). (Alday, 2008) Dapena and Feltner were able to estimate the air density value, \( \rho \), from the air density value, 1.225 \( \frac{kg}{m^3} \) at 15\(^\circ\)C found by Parker in his 1974 paper. (Dapena, 1987)

While the drag coefficient, \( C_d \), was found empirically and depends on the shape and surface properties of the physique and clothing of the athlete. Robert Walpert, from Schwinn Bicycle Company, and Chester Kyle, Professor in the Department of Mechanical Engineering at California State University, Long Beach, performed wind tunnel experiments on both a human and a life-like mannikin to find using linear regression in a running position the value of \( C_d \), which is constant across different velocities. (Alday, 2008)

The frontal area of the athlete, \( A \), which is thought to be proportional to the total surface area of the athlete \( A_b \), can be calculated from the equation:

\[
A = KA_b
\]  

\textsuperscript{20} The MATLAB function ode45 utilizes a Runge-Kutta method with a variable time step for efficient computation. ("Using ODE45")
In equation (20) \( K \) is the constant of proportionality. (Alday, 2008) Dapena and Feltner then found \( K \) to be equal to 0.24. Then \( A_b \) can be estimated from the mass, \( M \), and the standing height, \( h \), of an athlete using a modified DuBois equation derived by Vaughan and Sherwood:

\[
A_b = 0.217h^{0.725}M^{0.425} \tag{21} \quad \text{(Alday, 2008)(Dapena, 1987)}
\]

The values of \( M \) and \( h \) are based on typical height and mass values for world-class 400m athletes. (Alday, 2008) Substituting the values for \( M \) and \( h \) into the equation for \( A_b \), yields \( A_b = 2.1355 \). Then using the values for \( A_b \) and \( K \) gives the appropriate value for \( A \). (Alday, 2008)

The corresponding value for \( \alpha \) was found to be 0.00284. However, when trying slightly different \( \alpha \) values, such as 0.00282 and then analyzing the output of the MATLAB simulations it was found that \( \alpha \) is a sensitive parameter. Further investigation into this parameter will ensue throughout this project. In order to investigate \( \alpha \) properly, an analysis of the component parameters, \( \rho, C_d, A, h, M \), needs to be completed.

The MATLAB code was created so that times for corresponding distances at intermediate points on the track would be calculated for a total of 43.2s. The reason that 43.2s was chosen was because through trial and error it was found that this time produced a distance run of 400m or more for each lane. Then linear interpolation was used to find the times that corresponded to each of the following distances: 50m, 100m, 150m, 200m, 250m, 300m, 350m, 400m, and finishing time.\(^{21}\)

The model can also be employed for females using the following parameters: \( F = 7.14 \text{ m/s}^2 \), \( \beta = 0.00715 \text{ s}^{-1} \), \( \tau = 1.44 \text{s} \), \( \alpha = 0.0031 \), \( \rho = 1.184 \text{ kg/m} \) at 25\(^\circ\)C, \( C_d = 0.715 \), \( A = 0.43 \text{ m}^2 \), \( h = 1.71 \text{ m} \), \( M = 59 \text{ kg} \), \( K = 0.24 \) and \( A_b = 1.8113 \). (Alday, 2008)

\(^{21}\)Finishing time is the 400m time with 0.15s added to take into account the athlete’s reaction time. (Alday, 2008) 0.15s was found to be the average for world-class male sprinters, with reaction times faster than 0.13s being extremely rare. (Quinn, 2004)
3.3.2. MODELING STANDARD TRACK WITH AIR RESISTANCE

Simulations including air resistance followed the same general procedure that was discussed in section 3.3.1. However, now to measure wind effects, the wind speed becomes \( u = 2\sigma \), and the angle of the wind, \( \theta \), loops through each of the following wind directions: 30°, 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°. Thus, the effects of air resistance in different directions was able to be modeled. (Alday, 2008)

3.3.3. MODELING STANDARD TRACK WITH ALTITUDE

To allow for the effects of altitude for the IAAF standard track, the same general procedure that was implemented in 3.3.1. However, as explained previously, equation (10), is now used in place of \( \rho \) in order to model the air density \( H \) meters above sea level, and equation (11) replaces \( \beta \), since the propulsive force decays at a faster rate than at sea level. Thus, the parameter values that were used were \( g = 9.8, R = 287 J * kg^{-1} * K^{-1}, T = 25^\circ C, \rho_0 = 1.184 kg/m^3, \) and \( H \) loops through each of the following altitudes: 500m, 1000m, 1500m, 2000m, and 2500m. Then we solved for \( \beta \) with \( \beta_0 = 0.0006 s^{-1}, \sigma = 0.000023m^{-1} \), and \( \gamma = 0.35 \). (Alday, 2008) Thus, the effects of altitude were able to to modeled.

3.4. USING STANDARD TRACK MODEL TO MODEL THE DOUBLE BEND TRACK

Our model simulated the double-bend track in a similar fashion as was done for the standard track in windless conditions, with air resistance, and with altitude. Our model used the same parameter values for males used previously, as well as, the Euler method. However, now two differing radii had to be taken into account for each lanes’ bends, which were outlined in Table 2. Also, since the bends, straights, and radii are different lengths from the standard track a new set of equations for the relative wind velocity, \( v_w \), on different portions of the track had to be created. We recall that the double-bend track’s bends are each made up of three sections: the first with arc angle \( q \) and radius \( r \), the second with arc angle \( Q \) and radius \( R \), and the third with arc angle \( q \) and radius \( r \), where \( q = 70^\circ \) and \( Q = 40^\circ \), making this track geometry consist of eight sections.
Not only are the $v_w$ equations different from the standard track, but we also took a different approach when determining $v_w$. We first found the central angle, $\mu$, of the direction of the runner, and used this to give us the radial vector:

$$< \cos(\mu), \sin(\mu) >$$  \hspace{1cm} (22)

We then needed to find the tangent vector to this radial vector. Thus, we had to find the vector that when the dot product was taken between the two gave a result that was 0, and therefore, orthogonal. We found that the following tangent vector gave us that result:

$$<- \sin(\mu), \cos(\mu) >$$  \hspace{1cm} (23)

We can see that:

$$< \cos(\mu), \sin(\mu) > < - \sin(\mu), \cos(\mu) > = - \cos(\mu) \sin(\mu) + \sin(\mu) \cos(\mu)$$  \hspace{1cm} (24)

$$\Rightarrow 0.$$

We could have also chosen the following vector to be the tangent vector, however, it would have had the sprinter running in the wrong direction.

$$< \sin(\mu), - \cos(\mu) >$$  \hspace{1cm} (25)

It is this tangent vector (23) that we used as the direction vector of the sprinter. We then calculated the central angle, $\mu$, for each of the eight sections of the double-bend track, substituted them into (23), and called the result $S$. The following equations give $S$ for each of the eight sections of the track.

First Bend:

Section 1: $S = \langle - \sin((-\frac{240-3q-2QR}{r}), \cos((-\frac{240-3q-2QR}{r})) \rangle \hspace{1cm} (0 \leq s \leq 240 - 3qr - 2QR)$  \hspace{1cm} (26)

Section 2: $S = \langle - \sin((-\frac{3q}{2} + \frac{(r+240-2qr-2QR)}{r}), \cos((-\frac{3q}{2} + \frac{(r+240-2qr-2QR)}{r})) \rangle \hspace{1cm} (240 - 3qr - 2QR \leq s \leq 240 - 3qr - QR)$  \hspace{1cm} (27)

Section 3: $S = \langle - \sin(\frac{q}{2} - q + \frac{(r+240-3q-2QR)}{r}), \cos(\frac{q}{2} - q + \frac{(r+240-3q-2QR)}{r}) \rangle \hspace{1cm} (240 - 3qr - QR \leq s \leq 240 - 2qr - QR)$  \hspace{1cm} (28)

Back Straight: $S = \langle - \sin(\frac{q}{2}), \cos(\frac{q}{2}) \rangle \hspace{1cm} (240 - 2qr - QR \leq s \leq 320 - 2qr - QR)$  \hspace{1cm} (29)

Second Bend:

Section 5: $S = \langle - \sin(\frac{q}{2} + \frac{(r+2qr-2QR)}{r}), \cos(\frac{q}{2} + \frac{(r+2qr-2QR)}{r}) \rangle \hspace{1cm} (320 - 2qr - QR \leq s \leq 320 - qr - QR)$  \hspace{1cm} (30)
Section 6: $S = \langle -\sin(\pi - \frac{Q}{R} + \frac{(r-(320-q)r)}{R}), \cos(\pi - \frac{Q}{R} + \frac{(r-(320-q)r)}{R}) \rangle$ \quad (320 - qr - QR \leq s \leq 320 - qr) \tag{31}$

Section 7: $S = \langle -\sin(\frac{4\pi}{g} - q + \frac{(r-(320-q)r)}{R}), \cos(\frac{4\pi}{g} - q + \frac{(r-(320-q)r)}{g}) \rangle$ \quad (320 - qr \leq s \leq 320) \tag{32}$

Finishing Straight: $S = \langle -\sin(\frac{4\pi}{g}), \cos(\frac{4\pi}{g}) \rangle$ \quad (320 \leq s \leq 400) \tag{33}$

Next, we created a vector, $W$, which contains the wind component. This vector can be seen below:

$$W = < u \cos(\theta), u \sin(\theta) >$$ \tag{34}$

Finally we took the dot product of $W$ on $S$ to find the scalar projection of the wind onto the unit direction of the sprinter, which can be seen below.

$$v_w = W \cdot S$$ \tag{35}$

$$\Rightarrow u \cos(\theta)\sin(\mu) + u \sin(\theta)\cos(\mu)$$ \tag{36}$

Another alteration that needed to be made to the standard track model in order to model the double-bend track was the fact that $\lambda$, changes throughout each bend, due to its reliance on the radii. Thus, during sections one and three of each bend, $\lambda = \frac{rR}{v_0}$, and during section two of each bend, $\lambda = \frac{Rg}{v_0}$. This change in $\lambda$ throughout each curve, further causes $\omega$ to change during each section of each bend.

Now that the proper corrections have been made to our model to simulate the double bend track, we can run it and then compare the results to the standard track.

3.5. CASE STUDY OF POTENTIAL INTEREST

After creating a more complex model for the 400m and implementing it for different tracks, a case study of particular interest to pursue would be looking at the 2016 Rio Olympics men’s 400m final. This is because the WR was set with a time of 43.03s and Gold was won by Wayde van Niekerk of South Africa from lane 8. This would not come as a surprise since previous studies have shown that lane 8 is the fastest lane, however, this is the first time in history that the 400m has been won by an individual out of that lane at a major global track meet. Thus, the questions arise: why does this not happen more often, and why do top runners still prefer lanes 3-6, when they can run the fastest time from lanes 7-8? The answer
lies in that athletes at the top level do not necessarily race to run the fastest time, but rather to beat their competitors. Thus, psychological and strategic factors come into play, such as, athletes want to be able to see the competitors ahead of them and pace their efforts based on those other competitors. (Alday, 2010)

It would be interesting to use the mathematical model created to see if had Wayde van Niekerk been placed in a different lane, would his time adjusted for lane assignment, with all else being equal, still have won him Gold and a WR? Also, if an athlete from a different lane had been placed in lane 8 would their time adjusted for lane assignment, with all else being equal, instead, provided them with the Gold and a potential WR? Later on in our project we hope to address these questions and delve deeper into this record breaking performance.

CHAPTER 4
PRELIMINARY RESULTS

4.1. STANDARD TRACK RESULTS

The following three sections will outline the results arrived at from simulating Alday and Frantz’s model for the IAAF standard track. The first set of results applies to the standard track for windless conditions and zero altitude, and then the next two show the results taking into account air resistance and altitude, respectively. As the results will show, the model created in MATLAB provided very accurate and consistent results when compared to both Quinn’s model and Alday and Frantz’s model.

4.1.1. RESULTS OF STANDARD TRACK IN WINDLESS CONDITIONS

The first set of results outlines how our implementation compared to the Actual Time of male world-class athletes for each 50m intervals, Quinn’s Model Time, and Alday’s Model Time. Table 3, on the next page, details the comparison of each model for every 50m interval around the standard track for lane 4. It should be noted that the finish time is the time for the athlete at the 400 meter mark with 0.15s added on to account for the reaction time of the sprinter.

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22 The Actual Time was found by Ferro through the use of analogic video cameras, operating at 50 Hz, placed perpendicular to the running direction of the athletes when they passed each of the 50m sections around the track. (Ferro, 2001)
Table 3: Windless 400m Standard Track Model Comparisons. This table compares the times of world-class male sprinters for all the models for each 50m interval around the standard track for lane 4. (Alday, 2008)

As can be seen in Table 3, the results for our model were very similar to those arrived at by Quinn and more specifically Alday, whose model we tried to directly replicate. The biggest difference is at 150m, where our model produced a time of 15.88s, compared to Quinn’s 15.93s and Alday’s 15.98s, making our model 0.05 and 0.10s faster, respectively, in this instance. Other than in that specific case our model generally produced times that were 0.01s faster than Alday’s model. This phenomenon can be seen in the finish time for the Actual Time, Quinn’s model, and Alday’s model each producing a finish time of 43.18s, and our model producing a time of 43.17s.

Next, we show our model’s times for each of the 50m interval for each of the lanes. As shown in Table 4, each lane gets progressively faster, with lane 8 being the fastest. Thus, supporting the claim that the outside lanes are the fastest because of their wider radii and bends, while the inside lanes are the slowest because of their tighter radii and bends. Also, notice how our model predicted a lane 8 time of 43.03s, which is the same time that Wayde van Niekerk ran the WR in from the same lane.
Now that we know our model is accurate and consistent for windless conditions for the standard track, we can apply it to the air resistance and altitude cases.

4.1.2. RESULTS OF STANDARD TRACK WITH AIR RESISTANCE

In the previous section only the effects of track geometry on 400m race times were discussed. Now, the effects of air resistance will be considered. The first step was to compare our model’s output for a 2 m/s wind in each of the wind directions for lane 4 with those computed by Quinn and Alday. Table 5, gives the lane 4 400m times and time corrections for each of the wind directions for the three models. It should be noted that the time corrections are computed by subtracting the windless 400m time found in the previous section from the 400m time computed with air resistance and that the Time(s) and the Time Corrections(s) are rounded to the nearest hundredth, thus, the issue of having two close together but differing Time(s) might have the same Time Corrections(s), as well as, two close but differing Time Correction(s) might correspond to the same Time(s).
<table>
<thead>
<tr>
<th>Wind Direction (°)</th>
<th>Time (s)</th>
<th>Time Correction (s)</th>
<th>Time (s)</th>
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<td>+0.08</td>
<td>43.12</td>
<td>+0.09</td>
</tr>
</tbody>
</table>

Table 5: Air Resistance 400m Standard Track Model Comparisons. This table compares the times and time corrections of world-class male sprinters for all the models in lane 4 for a 2 m/s wind in each wind direction. (Alday, 2008)

Once again our model remains accurate when juxtaposed with Quinn’s and Alday’s models. In all instances our model gives time corrections that vary by only ±0.01 seconds or less from Alday’s model. Based upon our model for lane 4, the wind directions of 180°, 210°, and 240° are the most favorable for an athlete, while for Alday’s 210° is the most favorable and Quinn’s 240°. These winds blowing in the “west” and “southwest” directions produce times that are slightly faster than with windless conditions, an interesting finding since a 400m race is run on a closed circuit. Then for all three models wind directions of 0°, 30°, and 330° were the least favorable. These winds blowing in the “east,” slight “northeastern,” and slight “southeastern” directions produce the times that give the largest disadvantage to the athlete.
The winds that are blowing towards the west equate to tailwind in the beginning of the race and a headwind at the end, while the reverse is true for the winds blowing towards the east. Thus, this proves the idea that it is more of an advantage to a runner if they face a tailwind earlier in the race and a headwind later on, than to face a headwind at the beginning and a tailwind at the end.

Next, the times and time corrections using our model in each of the wind directions for all lanes were computed. The detailed results can be see in Table 6, on the following page.
<table>
<thead>
<tr>
<th>Wind Direction (θ°)</th>
<th>Lane 1</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
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<tbody>
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<td>Time (s)</td>
<td>Time Correction (s)</td>
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<td>+0.05</td>
<td>43.15</td>
<td>+0.05</td>
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<tr>
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<td>43.19</td>
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<td>330</td>
<td>43.21</td>
<td>+0.07</td>
<td>43.18</td>
<td>+0.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wind Direction (θ°)</th>
<th>Lane 5</th>
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<th>Lane 7</th>
<th>Lane 8</th>
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</thead>
<tbody>
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<td>Time Correction (s)</td>
<td>Time (s)</td>
<td>Time Correction (s)</td>
</tr>
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<tr>
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<tr>
<td>60</td>
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<tr>
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<td>43.00</td>
<td>+0.05</td>
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<td>120</td>
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<td>+0.01</td>
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<td>42.98</td>
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<tr>
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<tr>
<td>330</td>
<td>43.07</td>
<td>+0.09</td>
<td>43.04</td>
<td>+0.10</td>
</tr>
</tbody>
</table>

Table 6: Air Resistance 400m Standard Track Lane Comparisons. This table compares the times and time corrections of world-class male sprinters for all the lanes for a $2 \frac{3}{4}$ wind in each wind direction.
When comparing the effects of wind direction for each of the lanes it can be seen that all wind directions provide a disadvantage to a runner in lanes 1 and 2, with the greatest disadvantage coming from a wind direction of 0°. Then for lane 3 all the wind directions also provide a disadvantage, except for wind directions 210° and 240°, which produce times equivalent to those in windless conditions. Lanes 5, 6, 7, and 8 all follow the same trend as lane 4, with the most advantage coming from the wind directions in the range of 180° to 240°, and more specifically the greatest advantage at a wind direction of 210°. In the future, we could even break down these wind directions into even smaller increments, in order to find the exact direction giving the most advantage. Then the greatest disadvantages occurring at wind directions of 0°, 30°, and 330°. It should be noted that the greatest advantage afforded to a 400m runner out of all the lanes and wind directions is lane 8 with a $\frac{2}{3} m/s$ wind in the direction of 210°. Thus, with all else being an equal an athlete competing from lane 8 with that specific wind profile would have a significant advantage over the other athletes in the race, an advantage that should be considered for record setting purposes. However, we can also see that lane 8 also incurs the greatest disadvantage when the wind direction is between 0° and 120°.

The following figure, Figure 5, shows graphically the results displayed in Table 6.
Now that the effects of air resistance for the standard track have been discussed, the effects of altitude will now be presented.

4.1.3. RESULTS OF STANDARD TRACK WITH ALTITUDE

Finally, altitude was considered as a factor that could effect a 400m race on the standard track. As shown in Table 7, our model produced identical time corrections for a given altitude for every lane. For example, the time correction for an altitude of 500m was -0.07 for every lane, and the same was true for all the other altitudes. Thus, it is inferred that, with all else being equal, altitude affects every lane equally. Our model also showed that even though, athletes fatigue quicker as the altitude increases, their times actually improve.\textsuperscript{23} This can be seen by the average time correction for 500m being -0.07s and for 2500m -0.31s. As discussed previously, this is because the 400m is primarily an anaerobic event, while it is the aerobic energy system that is affected by the increase in altitude.

\textsuperscript{23} Times get quicker as altitude increases for at least the altitudes that we modeled. Intuitively, this cannot be a linear relationship, since at some point the altitude will be so high that the athlete would not be able to breathe. Thus, the athlete would either run slower or not be able to run at all. This point of no return is a topic that could potentially be explored in the future.
Table 7: Altitude 400m Standard Track Lane Comparisons. This table compares the times and time corrections of world-class male sprinters for all the lanes at each altitude.

As stated earlier since each lane has the same time correction for each of the altitudes, we see that the averages of the time corrections are these same values. The average time corrections for our model can be seen on the next page in Table 8. Table 8, also compares our time correction averages with those
of Quinn’s model and Alday’s model. As to be expected our model compares perfectly to that of Alday’s and only slightly off of Quinn’s.

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Quinn’s Average Time Corrections (s)</th>
<th>Alday’s Average Time Corrections (s)</th>
<th>Consol’s Average Time Corrections (s)</th>
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</table>

Table 8: Altitude 400m Standard Track Model Comparisons. This table compares the average time corrections of world-class male sprinters for all the models . (Alday, 2008)

Figure 6 provides a graphical representation of our model’s average time corrections for each of the altitudes.

Figure 6: Effect of Altitude on Standard Track Average Time Corrections. This figure compares the average time corrections of world-class male sprinters for each altitude.

Thus, it has been shown that altitude plays a significant factor in how fast a sprinter can traverse a 400m race, and therefore, should be taken into more consideration for record setting purposes.
4.2. DOUBLE BEND-TRACK RESULTS

In the previous sections we have seen the results of our model for the standard track. Now the next three sections will detail our model’s results for the double-bend track in the following cases: windless conditions, with air resistance, and with altitude. We will then be able to compare how the double-bend track impacts the performances of elite male 400m athletes in relation to the influence we have seen that the standard track has.

4.2.1. RESULTS OF DOUBLE-BEND TRACK IN WINDLESS CONDITIONS

We will begin our analysis with the effects of track geometry for the double-bend track in windless conditions. As with the standard track the double-bend track maintains the trend that lane 1 produces the slowest times with each subsequent lane getting faster, and lane 8 being the absolute fastest.

Table 9, below, outlines the times for each 50m interval of the 400m race for all of the lanes.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Lane 1 Time (s)</th>
<th>Lane 2 Time (s)</th>
<th>Lane 3 Time (s)</th>
<th>Lane 4 Time (s)</th>
<th>Lane 5 Time (s)</th>
<th>Lane 6 Time (s)</th>
<th>Lane 7 Time (s)</th>
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<tbody>
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<td>42.92</td>
<td>42.89</td>
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<td>43.19</td>
<td>43.15</td>
<td>43.11</td>
<td>43.07</td>
<td>43.04</td>
<td>43.01</td>
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</tbody>
</table>

Table 9: Windless 400m Double-Bend Track Lane Comparisons. This table compares the times of world-class male sprinters for all lanes for each 50m interval around the double-bend track.

From looking at Table 9, we can see that the times and specifically the finish time are slightly faster for the double-bend track than for the standard track. The time differences for the double-bend track are between 0.01 and 0.03s faster overall for each of the lanes than the standard track. For instance, for
lane 4 on the standard track we had a finish time of 43.17s, while for lane 4 on the double-bend track we had a finish time of 43.15s. The largest time difference was for lane 6, where the standard track gave a time of 43.10s, while the double-bend track a time of 43.07s. All the other lanes gave a time of 0.01 or 0.02 seconds faster on the double-bend track.

These findings fall in line with those of Quinn’s final model. Recall, that it was discussed previously that of all the tracks Quinn studied in this paper, the “double-curve 1” and “maximum radius” tracks were the joint fastest, with the standard track being slightly slower. (Quinn, 2009) The reason this is the case is because the average of each lane’s two radii of the double-bend track is slightly wider than the radius of each lane of the standard track.

4.2.2. RESULTS OF DOUBLE BEND TRACK WITH AIR RESISTANCE

Next, we will take a look at the results of the double-bend track when air resistance is factored in. The general trend in results for the double-bend track were similar to those of the standard track, however, there were some key differences that can be seen in Figure 10. Previously for the standard track we saw that lanes 1 and 2 provided a disadvantage in all directions, however, now for the double-bend track there are some instances where these lanes produce times that are equivalent to that of in windless conditions. These directions are 120° for lane 1 and 120° and 180° for lane 2, with lane 2 gaining a slight advantage from a wind at 150°. Next, for lane 3, we begin to see more wind directions that provide an advantage to the runner, recall that previously this did not occur until lane 4. For lanes 3 and 4 the directions providing the most advantage are 150° to 210°. For the standard track we recall, that lane 4 did not gain an advantage until the wind direction 180°. Then for lanes 5, 6, 7, and 8 the directions providing the most advantage are 180° to 240°, the same exact directions that were seen for the standard track. Then finally

---

24 Recall that in our model we refer to Quinn’s “double-curve 1” track as the double bend track.
25 It should be noted again that in Table 10, the Time(s) and the Time Correction(s) are rounded to the nearest hundredth, thus, the issue of having two close together but differing Time(s) might have the same Time Correction(s), as well as, two close but differing Time Corrections(s) might correspond to the same Time(s).
we notice that for all lanes the wind directions providing the most disadvantage to the athlete are 0°, 30°, and 330°.

Table 10: Air Resistance 400m Double-Bend Track Lane Comparisons. This table compares the times and time corrections of world-class male sprinters for all the lanes for a $2 \frac{4}{5}$ wind in each wind direction.

<table>
<thead>
<tr>
<th>Wind Direction (°)</th>
<th>Lane 1</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
<th>Lane 6</th>
<th>Lane 7</th>
<th>Lane 8</th>
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</thead>
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<td>43.09</td>
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<td>Time Correction (s)</td>
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<td>Time Correction (s)</td>
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<td>Time Correction (s)</td>
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We can then see the results of Table 10 graphically in the following figure, Figure 7. We notice how similar the results are to the ones that were derived for the standard track, however, we are able to see the key differences that were discussed above. The key differences that are particularly noticeable are how lane 1 and lane 2 now both skim the horizontal axis and lane 2 even dips below slightly.

![Figure 7: Effect of Direction of 2 m/s Wind on Double-Bend Track Time Corrections. This figure compares the time corrections of world-class male sprinters for all the lanes for a 2 m/s wind in each wind direction.](image)

We then conclude that the double-bend track is a slightly faster track for running than the standard track when taking into account the effects of air resistance. This is due primarily to the fact that the double-bend track provides directions that give lanes 1 and 2 an advantage, whereas the standard track did not.

4.2.3. RESULTS OF DOUBLE-BEND TRACK WITH ALTITUDE

Finally, we will take a look at the results of the double-bend track when altitude is included. As with the standard track, Table 11, shows that a given altitude effects each lane equally. Also, once again the most advantage is is afforded at 2500m, the highest altitude analyzed. Thus, reaffirming the idea that at least to a certain extent the higher the altitude the faster a 400m sprinter can run.
Table 11: Altitude 400m Double-Bend Track Lane Comparisons. This table compares the times and time corrections of world-class male sprinters for all the lanes at each altitude.

Next, the average time corrections at each altitude was found and are detailed in Table 12, on the next page. The interesting finding was that the average time corrections discovered for the double-bend track were the same as those found for the standard track. For instance, the average time corrections for standard track and the double-bend track at 500m were both -0.07 seconds. This is due to the fact that
even though the average radius for each lane of the double-bend track were slightly wider than the radius of each lane of the standard track, these small differences were not a significant factor when calculating the effects of altitude on performance. Thus, the radii of the two tracks were close enough together, that the most significant factor in analyzing the consequences of altitude on performance, was the altitude and its accompanying reduction in air density. This makes sense, since radius is not used in any of the equations that were created to model the specific effects of altitude.

<table>
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<tr>
<th>Altitude (m)</th>
<th>Double Bend Track Average Time Corrections (s)</th>
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<tr>
<td>500</td>
<td>-0.07</td>
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<tr>
<td>1000</td>
<td>-0.14</td>
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<tr>
<td>1500</td>
<td>-0.20</td>
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<tr>
<td>2000</td>
<td>-0.26</td>
</tr>
<tr>
<td>2500</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

Table 12: Altitude 400m Double Bend Track Average Time Corrections. This table shows the average time corrections of world-class male sprinters for the double bend track.

We can also visually see the findings expressed in Table 12 in the following figure, Figure 8.

Notice how Figure 8 is virtually the same as Figure 6.

Figure 8: Effect of Altitude on Double-Bend Track Average Time Corrections. This figure compares the average time corrections of world-class male sprinters for each altitude.
CHAPTER 5
CONCLUSION

So far this project has made progress into researching the effects of track geometry, air resistance, and altitude on various types of tracks, including the standard track and the double-bend track. Compared to previous models our model for the standard track was successfully implemented for windless conditions, air resistance, and altitude. We reaffirmed that, with all else being equal, the fastest lanes are those with the largest radii, that a tailwind gives the most advantage at the start of the race, that a headwind is best faced at the end of a race, and that for the altitudes analyzed an increase in altitude causes times to be faster. After, corroborating our results for the standard track to those previous models we were able to expand our research into a type of track that has only been briefly studied. We were then able to see the effects that track geometry, air resistance, and altitude have on the double-bend track and see how these results compared to those found for the standard track. We discovered that the double-bend track tends to be slightly faster than the standard track, because it has a wider average lane radii, the double-bend track has directions that give lanes 1 and 2 equal times to windless conditions or faster, and that the average altitude corrections for the double-bend track are the same as for the standard track.

All the research that has been done so far has laid the groundwork for what we would like to accomplish next semester. The next steps to this project are vital in order to get a better understanding of how a 400m race can be impacted by outside factors. In the future we want to investigate and better approximate the different parameter values that are used within the model, particularly $\alpha$ and those variables that compose it. This is because every sprinter’s body type is different, and it is unrealistic to only factor in body mass and frontal area in order to model this. Other body size measurements could possibly be included, such as body composition, Body Mass Index (BMI), and even leg length. We could then test to see which body dimensions or body types are the best in order to maximize 400m racing performance, and if these change depending on the lane or type of track the athlete is racing on. Next,
adding some other variables to the model might improve its predictive ability; these variables could include: humidity, the track’s surface, the spectator attendance at the race, or the importance of the race.

Also as stated previously, a case study analysis of the Rio Olympics men’s 400m final will likely ensue. In this we will see if Wade van Niekerk would have still broken the WR and won Gold had he run in a different lane, as well as, would another athlete been able to break the WR and/or win Gold had they run in lane 8.

Finally, we would like to find out at what point does altitude become a disadvantage to a 400m sprinter. We have seen that for the altitudes analyzed in our model an increase in altitude seems to improve performance, however, this can only be true up until a certain point. Once a certain altitude is reached the athlete will begin to have a hard time breathing, much less run fast. I would like to find out what this altitude is, which may or may not have to include adjusting the model a bit.

The 400m event is one of the toughest track races, as it mixes both strength and speed and the anaerobic and aerobic energy systems, however, it is the least researched of the sprint events. Its counterparts the 100m, 200m, and 4x100m generally take the spotlight when it comes to spectator excitement, ease of modeling, and belief that outside factors have a greater influence on performance. However, we have shown and will continue to show in the future, that the 400m can be modeled, outside factors do play a significant role in the outcome of this race, and that these factors need to be considered when allowing records for the 400m.


Dapena, J., and M.E. Feltner. "Effects of Wind and Altitude on the times of 100-meter Sprint Races."


Quinn, Mike D. "The Effect of Track Geometry on 200- and 400-m Sprint Running Performance."


