

# How Many Bridge Auctions?

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In normal games of bridge, only a small fraction of the possible auctions (that is, sequences of bids and passes) are ever used. One might therefore be surprised at the actual number of distinct auctions.

The game of bridge is played with four players comprising two teams, North-South, and East-West. We will assume that North opens the bidding, which proceeds clockwise. Each player bids or passes. The 35 possible bids are (in increasing order) 1 club, 1 diamond, 1 heart, 1 spade, 1 no trump; 2 clubs, . . . 2 no trump; . . . ; 7 clubs, . . . 7 no trump. Each bid must be higher than the preceding bid. The bidding terminates if all four players pass on the first round, or when 3 consecutive passes occur after the first bid. In place of a pass or bid, a player may "double" the last bid provided that this bid was not made by his partner. For example, South cannot double North's bid if East passes. Following a double, a player may "redouble" provided that he does not redouble his partner's double. There can be no more doubling of a bid after a redouble is made, unless a subsequent bid is made - in which case it too may be doubled and then redoubled.

**Theorem.** There are  $(4(22)^{35}-1)/3$  distinct bidding auctions.

**Proof:** There are 35 possible bids other than passes, doubles, and redoubles. Every auction involves exactly one subset of these 35 bids. Once the subset is determined, the order of the bids is determined.

Suppose we are given a subset of size  $b$  ( $0 < b \leq 35$ ). The first of these  $b$  bids may be preceded by 0, 1, 2, or 3 passes (four possibilities). In between each of the two bids, there are twenty-one possibilities: three in which no one doubles, six in which someone doubles but no one redoubles, and twelve in which someone redoubles. (Recall that one may not double or redouble one's partner.) After the last of the  $b$  bids, there are seven possibilities: everyone passes, or either opponent doubles followed by three passes, or either opponent doubles and either of the last bidder's team members redoubles. This provides the total of  $4 \times 21^{b-1} \times 7 = 4(21)^b / 3$  possible auctions, each of which involves precisely  $b$  bids. If  $b=0$ , there is one possible auction (everyone passes). Therefore, the total number of possible auctions is:

$$1 + (4/3) [(35! / 1! 34!) 21^1 + (35! / 2! 33!) 21^2 + \dots + (35! / 35! 0!) 21^{35}] = (4(22)^{35}-1)/3 \approx 1.29 \times 10^{47}.$$

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