Constant Neighbor Dihedral Tilings with 15, 32, and 43 Neighbors

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We consider tilings of the plane. Two tiles are called **neighbors** if they share at least one boundary point. A tiling is called a **constant neighbor tiling** if every tile has the same number of neighbors. A tiling of the plane is called **monohedral** if every tile is congruent, and a tiling is called **dihedral** if exactly two different tiles are used.

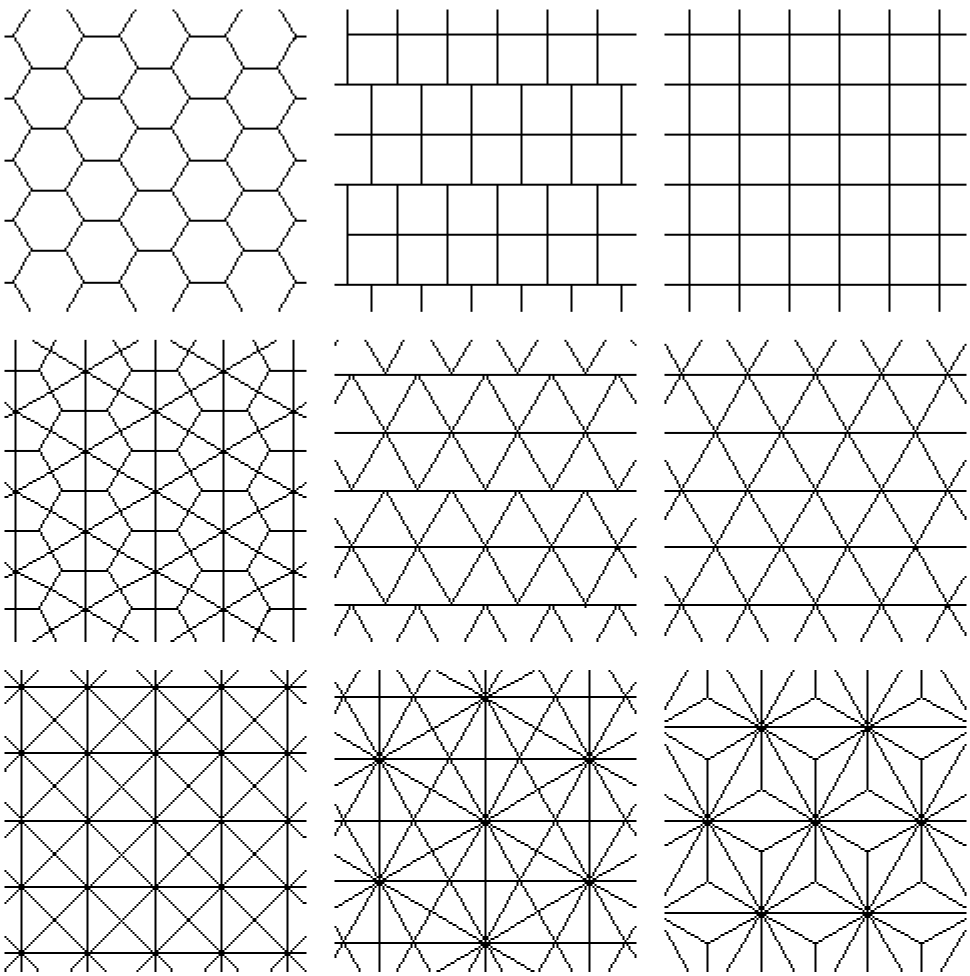

Figure 1. K(n)=1 for n=6, 7, 8, 9, 10, 12, 14, 16, and 21

L. Fejes Tóth defined K(n) as the least number of polygons whose congruent copies form a constant neighbor tiling. G. Fejes Tóth proved that K(n) exists for n > 5, and that K(n) ≤ (n+1) / 2. It is known that K(n)=1 for n=6, 7, 8, 9, 10, 12, 14, 16, and 21, and it is conjectured that these are the only such values. These monohedral tilings are shown in Figure 1.

The only other upper bounds known for K(n) were K(11) ≤ 2 and K(13) ≤ 2. Dihedral tilings with these n values are shown in Figure 2. We verify that K(15), K(32), and K(43) are all at most 2 by exhibiting such dihedral tilings. A dihedral tiling with 15 neighbors can be seen in Figure 3, and tilings with 32 and 43 neighbors can be seen in Figure 4. Clearly the same trick can be used to show K(22-12) ≤ n and K(22-1) ≤ n.

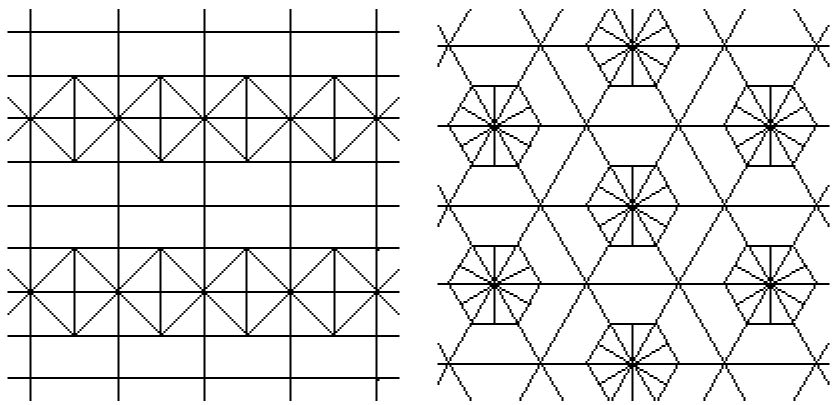

Figure 2. K(11) ≤ 2 and K(13) ≤ 2

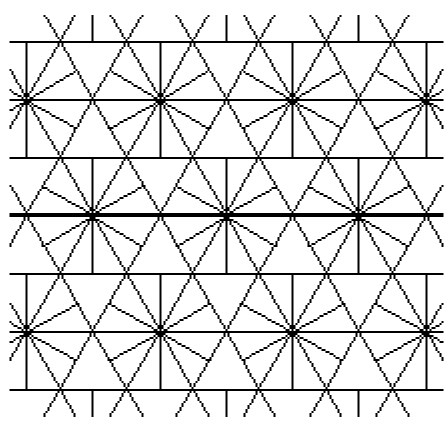

Figure 3. K(15) ≤ 2

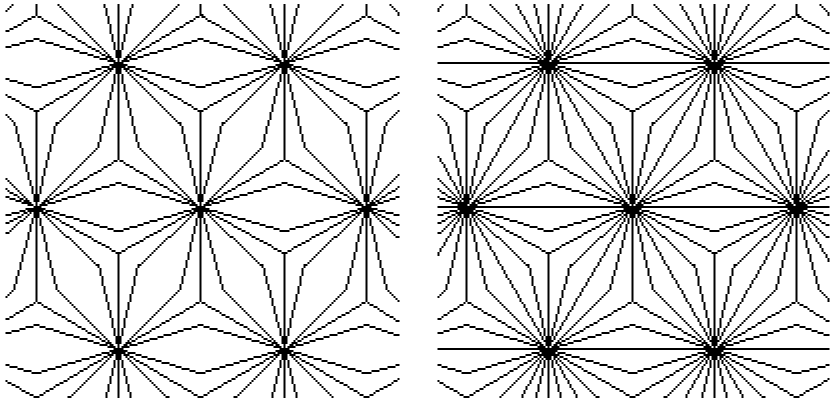

Figure 4. K(32) ≤ 2 and K(43) ≤ 2

G. Fejes Tóth was interested in this problem for convex tiles. Note that all the tilings we presented use convex tiles except for the last two. It is unknown how fast K(n) grows, or even whether K(n) --> ∞ as n --> ∞ .

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